Space Graph: Properties of 2-D Space in Paper Folding

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Submitted towards the fulfillment of the requirements for the Doctor of Architecture Degree

School of Architecture
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Doctorate Project Committee
Spencer Leineweber, Chairperson
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We certify that we have read this doctorate project and, in our opinion, it is satisfactory in scope and quality in partial fulfillment for the degree of Doctor of Architecture in the School of Architecture, University of Hawai`i at Mānoa.

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A subtle chain of countless rings

The next unto the farthest brings;

The eye reads omens where it goes,

And speaks all languages the rose;

And, striving to be man, the worm

Mounts through all the spires of form.

The rounded world is fair to see,

Nine times folded in mystery;

Though baffled seers cannot impart

The secret of its laboring heart,

Throb thine with Nature's throbbing breast,

And all is clear from east to west,

Spirit that lurks each form within

Beckons to spirit of its kin;

Self-kindled every atom glows

And hints the future which it owes.[1]

[1] Ralph Waldo Emerson, *The Essential Writings of Ralph Waldo Emerson*, (New York: The Modern Library, 2000).

ABSTRACT

This doctorate project thesis explores space within the context of flat paper folding, spatial systems, and geometry in a 2-Dimensional format. This thesis considers that space acts as an interdependent feature with the configurations of physical matter. The term “paper folding”, rather than *origami*, is used as the preferred term. Although both concepts are considered interchangeable by the paper folding community, origami has specific cultural associations which are outside the scope of this thesis. This thesis shows how the information of space is structurally contained in the layering process in 2-Dimensions of paper folding. Space can be visualized into chunks or parts not by slicing space physically but by understanding it through folding. The properties of 2-Dimensional space are explored in the flat paper folding process by visualizing the folding cycle through concyclic graphs and hyperplanes. The exploration is primarily conducted through a nominal size, single isometric square sheet of paper, which cannot be cut nor glued, and must be flat-foldable into 2-Dimensional form. The graphs are taken through the entire paper folding cycle. As the layering process is of primary importance, a translucent paper medium is used track changes during folding. The primary basis for this thesis is that space and physical form, or object, share an interdependent relationship. The experiment is based on three core ideas. First, space can be represented physically through geometry as a line, more specifically, a hyperplane. Second, space and the object represented in the folding process must remain isometric or continuous, and cannot be physically cut. And third, paper has two operative sides, one positive (front) and the other negative (reverse). The research for this thesis is based on contemporary literature and theory, and acknowledges the possibility that other relevant and valid knowledge of space may exist from other sources. There are no special facilities or equipment which is used for this work, nor does it require any human or biological subjects, or samples for study.

One method proved particularly helpful in organizing many of the initial questions related to the research problem, TRIZ Analysis. TRIZ is a step-by-step methodology specifically developed to address ill-defined topics. Genrich Altshuller the developed the method of TRIZ (*Teoriya* ***R****esheniya* ***I****zobretatelskikh* ***Z****adatch)* or the Theory of Inventive Problem Solving during the 1950s.[2] Altshuller discovered that patterns of technological development exist in virtually known scientific and technical discovery. Altshuller discovered that the patterns of technical-knowledge followed a time-dependent process, where both the entire life-cycle of technological development or scientific ideas could be taken into account and could also be used to predict its next stages. The generalized principles on which TRIZ is based can be applied to virtually any field of study. Although the methodology is currently used mainly in the science and engineering fields, there are several examples which TRIZ is used in architecture. Examples include those from Joe A. Miller[3], Darrell Mann's Computer Based TRIZ-Systematic Innovation Methods for Architecture[4] and 40 Inventive (Architecture) Principles with Examples.[5] For a more detailed description of TRIZ the reader may consult the TRIZICZS reference. To determine where each area of study is plotted along the evolutionary graph their technological states are compared through the list of evolutionary steps listed below. As this may be a first application of TRIZ to paper folding and space, the conclusions are basically observations based on experience during the doctorate thesis project.

The Evolutionary graph, Figure A is based on 5 time-dependent stages where any subject can be graphed and compared with other related studies. The graph is created from a selection of 40 technological patterns, based on physical behavior, 21 of which were pre-selected for this analysis. The closest area of study, at least in architectural terms, the context of which this thesis is written, is

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[2] Gordon Cameron. TRIZICS, CreateSpace, 2010.
[3] Joe. A Miller. *TRIZ Solutions* for *Systems Dynamics Models of A Small Community Downtown Revitalization Project.* TRIZCONN 2004, Seattle, 2004.
[4] Darrell Mann and Connal O Cathain. *Systematic Innovation Methods For Architects.* University of Bath, Bath: 2001.
[5] Darrell Mann and Connal O Cathain. *40 Inventive (Architecture) Principles with Examples.* University of Bath, Bath: 2001.

an area of spatial analysis called space syntax. The pink graph describes general spatial analyses, the green shaded area shows the space syntax plot, while the purple area show the graph of paper folding. The white areas indicate untapped potential of all respective fields. The Evolutionary Graph shows that the areas of highest potential of the paper folding model include object segmentation, mechanic substitution, and color changes.

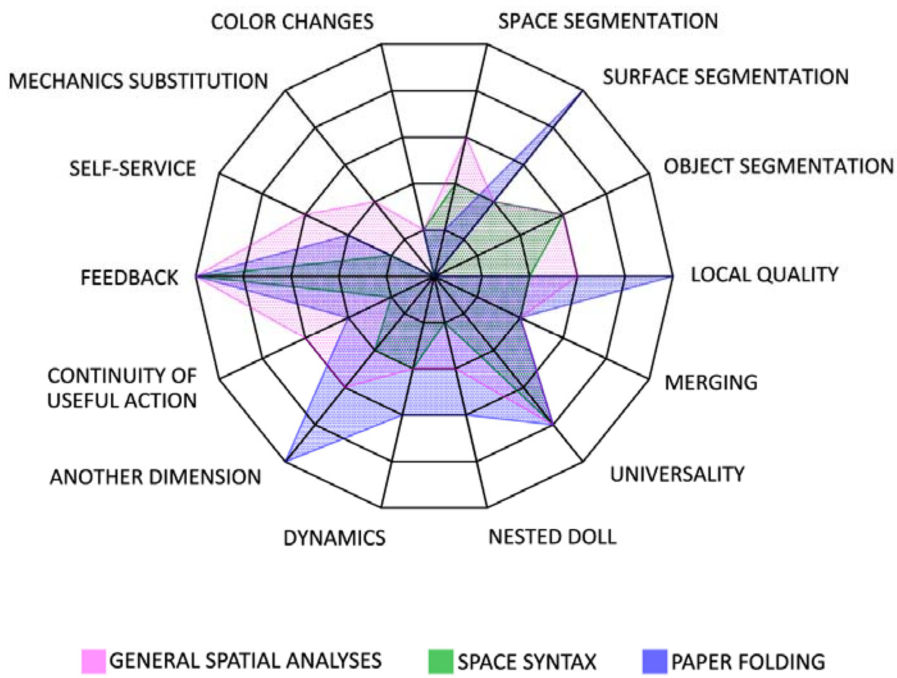

Figure A., TRIZICS Evolutionary Graph[6]

There are five areas of potential in terms of how the research problem might be approached, these include: 1) space segmentation, 2) object segmentation, 3) mechanics substitution, and 4) color changes. It was determined that a mirror line or hyperplane could be used in combination with the features identified in the Evolutionary graph to approach the thesis problem.

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[6] Graph by author.

ACKNOWLEDGEMENTS

This work is dedicated to all those who have come across my journey both past and present, and to those who I have yet to meet. The invaluable lessons that I have learned on my academic journey, I believe, do inform how we treat the people and the world around us. I would also like to recognize Ms. Irene Moir of the John and Gertrude Moir Educational Fund Scholarship, which has made some of this work possible. Patience is one of the most gracious gifts anyone can receive, it is a debt I can never repay back to my family, but hopefully this serves as a token for their generosity and love.

Space Graph: Properties of 2-D Space in Paper Folding

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SPACE GRAPH

ch1

INTRODUCTION

1. INTRODUCTION

In general a lot is known about buildings, their construction, design, material, and use are well understood but little is known about the nature or operative behavior of space. Space has an intimate and enigmatic relationship with the physical world. Although space does not appear to have any physical qualities or directly visible structure, there is a clue to its behavior through the distribution of patterns in the natural and human-created environment.[7] Space is found in a variety of phenomena such as the formation of black holes; to the symbiotic biology and formation of a coral reef; to the patterns formed by the ocean on a sandy shore; to the dynamic growth and change of cities; to something as seemingly mundane as the creation of a bird's nest. Space is perceived in conjunction with physical form as a kind of thread linking experiential and functional phenomena. Space appears to define things which seem completely separated through both physical distance and time. This suggests that space not only has order but has a sequence of local and global behaviors, and therefore structure, which can be observed and visualized—like following a string through a maze. There is a large body of literature from the arts, sciences, and philosophy dedicated to the topic of space. In the sciences geometry is used to study the behavior of space, but its true nature remains elusive. Space is also formally, but indirectly, studied in architecture by means of space syntax analysis. Experts in the fields of science and mathematics have suggested that space could be physically cut into slices. The concept of cutting space into parts is also familiar to architecture, but as a practical function. It forms a fundamental means to convey building information through plans, sections, and perspectives. According to Graham Nerlich space is perceptible in principle and that it is its geometric type which makes it elusive in a non-causal way.

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[7] Graham Nerlich, *The Shape of Space* (Great Britain: Cambridge University Press, 1994), 1.

SPACE GRAPH

ch2

SPACE

2. SPACE

> Space is an order of coexisting phenomena [*spatiun fit ordo coexistentium phaeonenorum*]. (Gottfried Wilhelm Leibniz)[8]

> "Originally, 'space meant literally 'to determine boundaries'; space had rarely been discussed before the beginning of the twentieth century. It is not the symptom of professional naivety or economic ignorance, but a fundamental question which lies in the very nature of architecture and of its essential element: space." (Bernard Tschumi *Questions of Space*, Avant-garde theorist and Deconstructivist) [9]

The primary aim of this chapter is to explore how space is a physical entity, is understood as a system, and how it can be studied through the regularized patterns found in paper folding. This chapter covers three areas: 1) historical and modern views of space, 2) spatial systems such as space syntax used in architecture, 3) and the developments in the paper folding phenomenon.

The concept of space as philosophical discourse can trace its origins to classical Greece where the ideas of *space* and *place* are parallel concepts. The concept of space as it developed from antiquity was founded on Euclidean mathematics for which space, as a continuum with its own independent reality, was never fully posited; the elements of which this system was constructed--the point, the line, and the plane---were nothing more than idealizations of solid bodies; Space itself

[8] Gottfried Wilhelm Leibniz, Die Philosphischen Schriften, ed. C.J. Gerhardt (Berlin: 1875-90).
[9] Bernard Tschumi, *Questions of Space: Lecture on Architecture* (London: E.G. Bond, 1990), 12.

emerged only secondarily, that is, only insofar as it could be derived from these idealized forms and the relations produced by their contact--intersections, points lying on lines or planes, etc.[10] Descartes (1596-1650) further liberated space as an infinite and independent reality with the Cartesian coordinate system.[11] Although the originator of the terms are not known, both concepts are discussed by Aristotle's predecessors such as Plato (424-328 B.C.E.)[12] in Timaeus.[13] In *Physics*, Book IV, Delta (4thc BCE), Aristotle (384-382 B.C.E.)[14] develops the ideas further by metaphorically linking place with particulars of location; and space, relating to the containment and delineation of it by physical objects.[15] Aristotle rejected the general idea of space as a void, and developed only a theory of positions as related with ideas of place. Aristotle's theory of places is of greatest pertinence not only because of its important implications for physics, but also because it was the most decisive stage for the further development of space theories.[16] As a result place and space developed along different paths, where the concept of place remained less developed over time. However, the concept of space experienced a significant conceptual leap when it was associated with infinity. Until the 11th century, space retained metaphysical assumptions. In the 11th century, Arabic scholars rediscovered Greek literature on the topic of space. A polymath named Ibn Al-Haytham, or Alhazen, applied the concept of space through the rigors of an objective methodology, or the scientific method, which he is often credited with developing its foundational ideas.[17] He related space to geometric configurations of the physical environment in *Discourse on Place* (*Qawl*

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[10] Sanford Kwinter, *La Città Nuova: Modernity and Continuity.* ed. K. Michael Hayes. *Architecture Theory Since 1968* (Colombia University and the Massachusetts Institute of Technology, MIT Press, 1998), 14-17.
[11] Analytic Geometry, Encyclopædia Britannica, Last modified August 2011, http://www.britannica.com/EBchecked/topic/22548/analytic-geometry.
[12] David Sedley, *Plato's Cratylus*, (Cambridge: Cambridge University Press, 2003).
[13] Plato, *The Timaeus of Plato.* (New York: Arno Press, 1973).
[14] Ingemar Düring, *Aristotle in the Ancient Biographical Tradition* (Göteborg, 1957), 253.
[15] Aristotle, *Aristotle's Physics, books III and IV / translated with notes by Edward Hussey (New York:* Oxford University Press, 1983).
[16] Max Jammer, *Concepts of Space: The History of Theories of Space in Physics. (Cambridge:* Harvard University Press, 1969).
[17] Muhammad Saud, *The Scientific Method of Ibn al-Haytham (*Ph.D.Thesis, Karachi: The Islamic Research Institute, 1990).

fi al-Makan). Although Alhazen did not prove the nature of space, he did manage to show its relationship with the physicality of the material world, at least in terms of a spatial concept.[18]

It would not be until the 17th century that European scholars took to the helm of science that space took its next conceptual leap. Two defining methods were developed with space: Absolute, Euclidean space as described by Sir Isaac Newton and the idea of relativistic space developed by his contemporary, Gottfried Wilhelm Leibniz (1646-1716). Absolute space, in its relationship with the Cartesian coordinate system proved overwhelmingly productive in describing the physical behaviors of the environment such as gravity and inertia. In contrast, Leibniz, another polymath, who developed calculus independently of Newton, described space as relativistic, where space and physical form are seen as interrelated parts.[19] This idea appears in a surprising way as discussed in chapter 4, Section 4.3. Although thematically related to the Relativity theories of the 20th century, they are based on unique arguments, a comparison of which is mostly outside the intended readership of this thesis. With such discoveries, however, space remained an ill-defined phenomenon.

In Physics, space is a fundamental quantity which cannot be described by other quantities of the environment. However, space has a relationship to other fundamental features such as gravity which can be measured.[20] Although this thesis does not define the cause of space, its known properties as described through the sciences share a deep relationship with how space is experienced and can be analyzed through architecture. Space is found in concurrent relationship with physical features of the environment, where it is not an acute phenomenon, but one which is consistently found in the environment regardless of scale or dimension. The terms of space and space-time are distinct. Space is generalized as a boundless phenomenon, and in conjunction with

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[18] Plato, *Timaeus*, trans. William Heinemann (Cambridge: Harvard University: Loeb Classical Library, 1961).
[19] Andrew Benjamin, *Architectural Philosophy* (London: The Athlone Press, 2000).
[20] Max Jammer, *Concepts of Space: The History of Theories of Space in Physics* (Cambridge: Harvard University Press, 1969).

time, forms a 4th dimension. This is one of the areas where early 20th century architects misinterpreted the application of space-time as a blanket theory for how one experiences space, which made it initially difficult to bridge architecture with objective qualities of the built environment. As a contemporary and close friend of Einstein, Kurt Friedrich Gödel (1906-1978) theorized that space and time are structurally different forms of matter.[21] Furthermore, Werner Heisenberg (1901-1976) stated that what we experience as the environment is actually a field of potentials, rather than objects.[22] While the scale of approach is outside this thesis problem, this thesis embraces idea that the physical environment can be described through the geometry related to fields.

Physics and science, in general, are particular ways of seeing. While this appears a hard and analytical description of space, space also appears as a feature that is 'soft' and malleable, but no less concrete or physically describable feature. Space is a concept that shares a continuum throughout human experience whether it takes the form of looking up at the stars in the cosmos through a backyard telescope, navigating the busy streets and narrow thoroughfares of a bustling city, or witnessing a live artist painting on one of those city's boardwalks. Space is a persistent feature in the human condition regardless of study, living condition, and scale. Noted geographer Yi-Fu Tuan discusses such multi-functional qualities of space. In *Space and Place,* he states: "…space can be experienced as the relative location of objects or places, as the distances or expanses that separate or link places, and,--more abstractly—as the area defined by a network of places."[23] The concept is also evoked in more subtle ways such as in dreams and emotions where space is often faithfully created, at least in terms of dimensions. Space is present in games of strategy such as chess

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[21] Palle Yourgrau, *A World Without Time: The Forgotten Legacy of Godel and Einstein* (Cambridge: Best Books, 2006).
[22] Werner Heisenberg, *Physics and Philosophy: The Revolution in Modern Science (*New York: Harper & Row, 1962).
[23] Yi-Fu Tuan, *Space and Place: The Perspective of Experience (Minneapolis:* University of Minneapolis Press, 2003).

or tennis where the activities take place through a constrained set of conditions. In the games of strategy, practice and time are manifested as proficiency in the activity. Upon closer examination of the examples given above, space appears to have an organizational feature, albeit an ambiguously defined one.

This idea makes it easier to consider space as a physical feature that can be understood in a more broad context, including the research topic of this thesis, paper folding. In a general sense, while it is difficult to bridge the large scale of the material environment with such unseen “forces” at the macrocosmic or microcosmic scale, space is a foundational concept in architecture and shares that continuum. It is the conjecture of this thesis that architecture necessitates a deeper understanding of space, other than as a given feature which a user experiences and the architect assumes to design. Perhaps some form of this problem can be approached in a humble manner through paper folding.

Paper folding and mathematics share a continuum. While exhibiting an emotive and directly immersive experience, paper folding is also a highly organized, objective, and precision based geometric activity. Paper folding is conceptually simple but its behavior is configurationally complex. Paper folding is both a soft and hard process. In its physical context, paper folding exhibits a remarkable range of complexity from single-fold forms, multi-part polyhedral constructions, to sophisticated crumpled, and textured objects. In its abstract context, paper folding shares relationships with fractals, algorithms, and information theory. This thesis explores both physical and abstract qualities of paper folding in the context of space. A typical example made by the author is shown below in Figure 2.1.

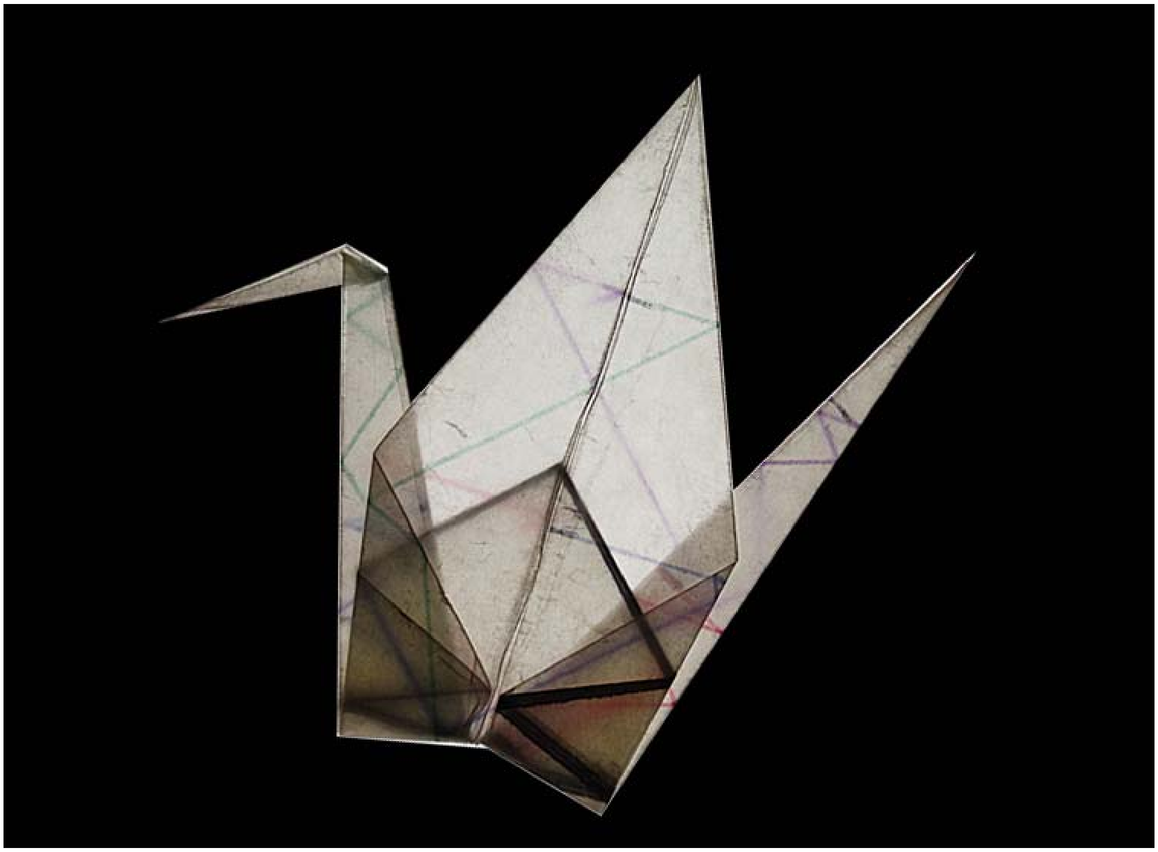

Figure 2.1. Paper Crane, 6”x6” wax paper, art and photo by author[24]
This is a case study using hyperplanes or mirror lines to visualize spatial configurations.

As a material, the invention of conventional paper is attributed to Tsai Lun of 11th century BCE China. As a tool for spatial communication, the earliest known existing folded map, called the Turin Papyrus, locating precious metal mines from historic Nubian Egypt, is dated to 1,150 BCE.[25] In the Middle Ages, both Oriental and European papers, the latter’s manufacture based on that of the former, were distinguished by a firm, stout substance; and in the laws of King Alphonso X of Spain (1252-1284) paper is referred to as cloth parchment.[26]

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[24] Fold and photograph by author, 2012.
[25] James A. Harrell, *Turin Papyrus Map from Ancient Egypt* (Toledo: University of Toledo, 2000).
[26] Samuel Randlett, *The Art of Origami.* (New York: E.P. Dutton & Co., 1961).

There are several parallel developments of paper folding found throughout Europe and the Middle-East as a result of Silk Road trading. Leonardo Da Vinci's treatise, *Codex Atlanticus*, contains a series of paper fold objects, most notably an airplane which resembles the modern variety. Da Vinci also made use of the term 'falcata' meaning 'bent' or 'folded'.[27] Throughout its history, paper folding served not only as a as an artisanal practice but functioned as a conceptual tool, as in Da Vinci's case. Japan developed paper folding to its highest form for several centuries after it was adopted from China. The earliest known paper ceremonial folding manual originates from Japan called *Tsusumi Musibi no* Ki by Sadatake Ise in 1764. Following that, in 1797 in Japan, was *How to Fold One Thousand Cranes* on recreational folding.[28] Akira Yoshizawa (1911-2005) is considered the founder of modern origami. For most of its existence, paper folding lived an esoteric existence used mainly as a hobby. Yoshizawa ushered in a new stage in origami practice—he raised it to the level of an artform. As a technical draftsman, Yoshizawa used origami to communicate geometric problems. The key in popularizing paper was the development of a standardized notation system which a layperson could interpret.[29] Samuel Randlett adopted the basic notation developed by Yoshizawa and organized it into the Yoshizawa-Randlett system of notation.[30] The instructional diagram system adapted in Figure 2.2 is the one which most people are familiar with through origami literature. Until the 1970's, it was generally thought that paper folding had no outside application, except in special cases of mathematics. Of special interest is what the diagrams describe in terms of the global properties paper folding transformations, rather than simply looking at the diagram in terms of instructions. In the context of this thesis the diagrams in Figure 2.2 can be interpreted as general transformation operations in which the information in the folding process is converted from 2-D to 3-D and 2-D

[27] Samuel Randlett, *The Art of Origam,* (New York: E.P. Dutton & Co., 1961).
[28] David Lister, "Errors and Misconceptions About the History of Paper Folding." Last modified September 3, 2011, British Origami Society, http://www.britishorigami.info/academic/lister/errors.php.
[29] Robert J. Lang, *Origami Design Secrets: Mathematical Methods for an Ancient Art* (Natick: AK Peters, 2003).
[30] Samuel Randlett, The *Art of Origami*, (New York: E.P. Dutton & Co., 1961).

again. The concyclic or shell graphs in Chapter 5 use these principles for the fold examples. Paper folding is a highly active spatial transforming activity, but many of its behaviors are subtle and hidden. There are subtle and complex transformations in paper folding which are not studied here. In the research the reader will notice that the patterns in paper folding can be regularized. If so, then there is the potential to study space on the basis of such research.

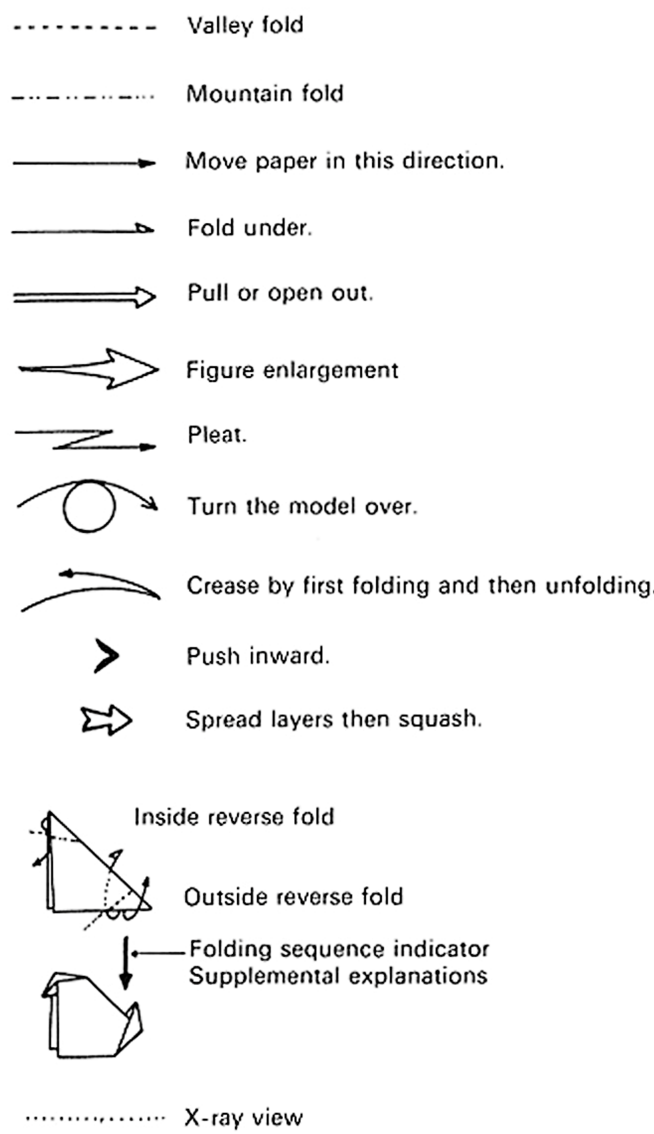

Figure 2.2, Yoshizawa-Randlett Paper Folding Notation System[31]

[31] Kunihiko Kasahara, *Origami Omnibus, (Tokyo:* Japan Publications, 2000).

Paper folding is a rich practice, applicable to many areas of study from art, to engineering, to mathematics and biology. The study covers a range of folding types from technical folding to artisanal wet-fold varieties. Although the hand-making quality remains an important aspect in paper folding, this thesis is primarily interested in the geometry related to its folding behavior. The intent is not to suggest that paper folding or space are purely mechanical, rather it is to say that the inner-workings of the folding process can be regarded as one of beauty, subtlety, and practical usefulness.

Robert Lang decomposed the problem of paper folding into a series of lines and circles called the Circle-River Method (CRM). In 2-Dimensional space, a type of folding method called the Circle-River Method is analogous to circle packing in geometry. In Figure 2.3 the Circle-River Method is a means to organize and distribute area within a limited single sheet of paper for specific parts to be folded. Through the CRM lines represent crease patterns, while the circles identify the areas that eventually form the physical features of the finished model. Lang adapted the idea into a computer program called Treemaker which allows a user to draw a representative stick-figure shape of any object or animal where the software then draws out a circle-crease folding pattern. Chapter 3 explains how the C-R Method shares a great deal of similarity with the Axial Line and Depthmap methods in terms of geometric spatial distribution of physical form.

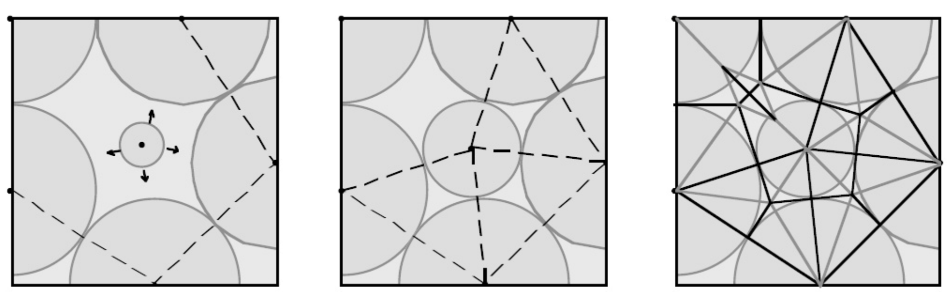

Figure 2.3. Paper Folding Circle Packing designed in Treemaker Software[32]

[32] Robert Lang, "Treemaker Software Crease Patterns," last modified December 2011, http://www.langorigami.com/science/computational/treemaker/treemaker.php.

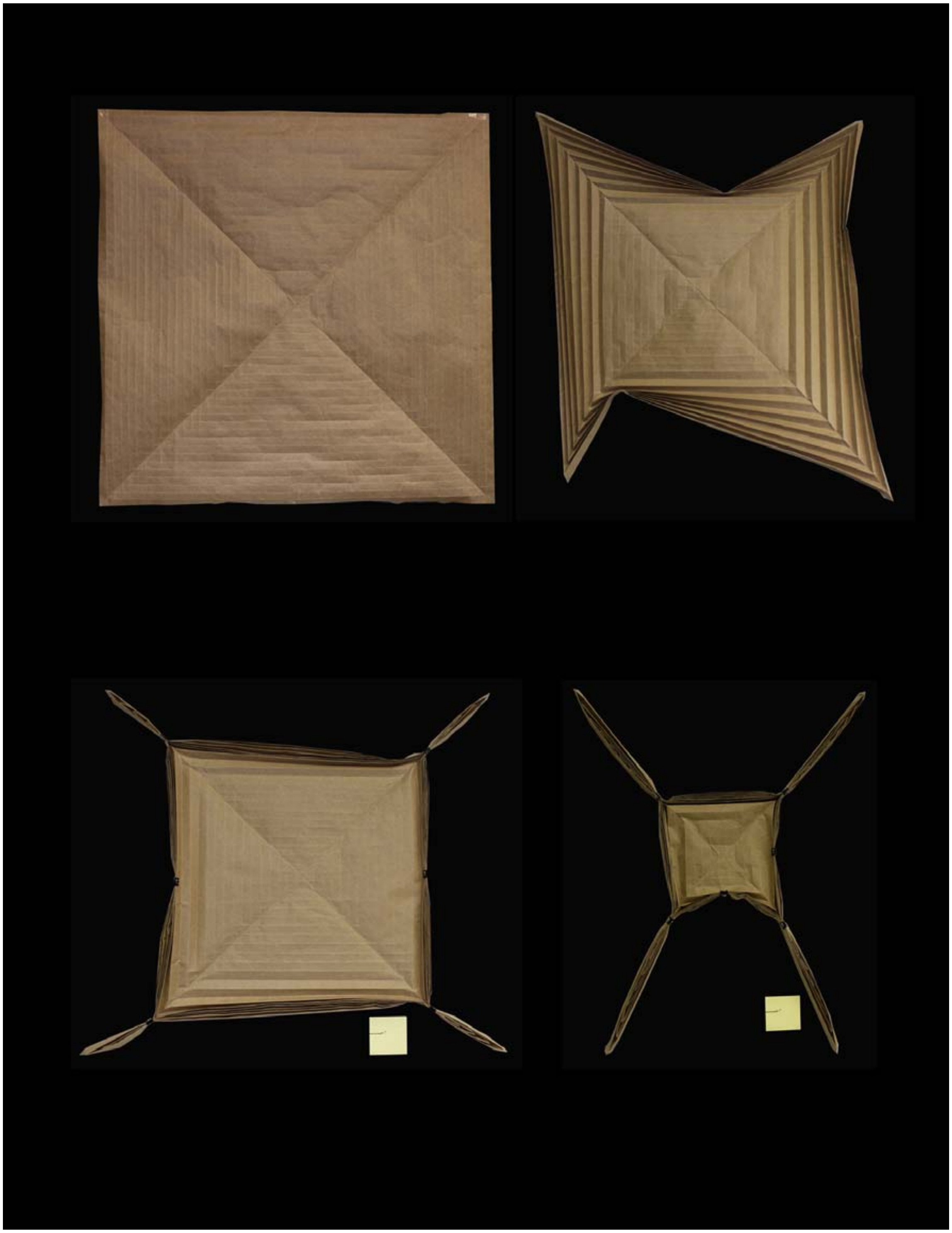

Figure 2.4. Parabola Fold, Postal Paper, 3’x3’[33]

[33] Art and photo by author.

Figure 2.5. Parabola Fold, Postal Paper, 3’x3’[34]

[34] Art and photo by author.

Paper folding is one of the most intimate activities related to space where several important features useful for this thesis are known. Paper folding is a flexible medium that is not merely a result of its material but due to a synergy between the interactions of self-imposed constraints. In and of itself, paper has no inherent rules. It is physically accessible while conceptually abstract. Simply from the malleability of the material alone one can intuit ideas of folding through objects such as newspapers, brochures, and letters. It is when the folding process begins that the sequential nature of paper folding becomes apparent. It is an applied physical application and art where many of its properties are not readily predicted by planning or analysis. Although paper and paper folding have existed for several centuries alongside mathematics, it is probably one of the reasons why the activity existed more as an artistic practice than an analytical one, simply due to one of practical application between the areas of study. Paper folding is not solely the domain of spatial relationships and geometry, but internalizes emotive characteristics associated with aesthetics and art. Paper folding is a powerful tool because it integrates spatial configurational characteristics and ideas of proportion and structure, which can be useful for a designer and the artist.

The first historic documentation of studying paper folding with mathematics comes from the work of Norbert Wiener and T. Sundara Row who both explored the idea independently. Wiener's work is undated, but T. Sundara Row's is dated to 1893 in India.[35] Twelve years later *Solving Cubics with Creases* was published by Margherita P. Beloch from the University of Italy in 1905. Beloch identified analytical properties of paper folding including: emergent combinatorial properties, mappings of Euclidean space, and mathematical geometric constructions. A Renaissance in paper folding began in the 1970s when a comprehensive understanding of the inherent geometric characteristics would be found in paper folding. The recent ground-breaking technical work has come from physicists, computer scientists, mathematicians, and even high school students.

[35] "On a Transformation by Paper Folding*." The American Monthly, Vol 31, No.9*, November 1924, 432-435.

Researchers such as Robert Lang and Erik Demaine made several significant finds dealing with the patterns of complexity found in paper folding. The mathematical literature regarding paper folding has vastly expanded in the last 20 years.

Space, geometry, and mathematics are present in paper folding in surprising ways. While paper folding is generally a 3-Dimensional phenomenon, many of its aspects are descriptively otherwise. Paper folding shares the same dilemma with the sciences and architecture that space, as an integral feature of the paper folding, remains little understood. However, paper folding has a unique set of tools which are helpful for the research problem. The most useful one here is its transformative ability in 1-D, 2-D, and 3-D discussed in Chapter 4 where the information of the transformations can be collapsed into a graph. It has been shown that paper folding can reproduce all geometric constructions, made by compass and ruler, in Euclidean space by folding alone.

A sheet of paper can be folded into a generic star pattern called a Koch snowflake. If a paper were infinitely thin but of finite size dimensions, a sheet can be folded to where it has an infinite perimeter.[36] [37] Although fractals are outside the scope of this discussion, fractal patterns of space appear in both the large scale built environment of cities and in complex natural patterns of the environment such as the formation of coastlines. For a comprehensive discussion on the topic of cities, the reader may look into *Cities and Complexity*[38] by Michael Batty. Batty made significant contributions to the development of the computer simulation software used in space syntax analysis. For further theories on the topic, the reader may also refer to the work of Gottfried Leibniz, who introduced the idea, and discussed in Clifford Pickover's *The Math Book: From Pythagoras to the*

[36] Margaret Wertheim, "The Mathematics of Paper Folding: An Interview with Robert Lang." Interview, Cabinet, Issue 17, Spring 2005.
[37] John Bryant, *How Round is Your Circle?* (New Jersey: Princeton University Press, 2008), 147-148.
[38] Michael Batty, *Cities and Complexity: Understanding Cities with Cellular Automata, Agent-Based Models, and Fractal* (Massachusetts: The MIT Press, 2005).

57th Dimension, 250 Milestones in the History of Mathematics.[39] Also of interest may be Benoit B. Mandelbrot's *The Fractal Geometry of Nature.*[40]

Since mathematics has only recently been applied in a comprehensive way in the last 30 years, many unforeseen discoveries are being made that have widespread application. Often when mathematics and geometry are concerned, the final folded object is half the discovery, where the abstract derivations discovered in the process proves just as valuable. A typical challenge brought up in many common paper folding discussions concerns the amount of times paper may be folded. Although such examples are outside the core concern for this thesis it does bring into awareness the mindset and qualities of mathematical argument, at least in paper folding. In 2005, a student by the name of Britney Gallivan showed that a single sheet of paper could be folded 12 times and developed a limiting equation proving that limit.[41] Like most mathematical problems, there are certain conditions which Gallivan set as parameters, specifically that the paper is of a certain thickness, rectangular in shape, unique layers are in a straight line, a length of 4,000 ft (1,200 m), and folded in one direction, rather than alternately. Gallivan's attempt has also been been proven independently. From this simple example, one can see that discussions in paper folding can take on a technical or mechanical mannerism. Due to the area of study from which this thesis is written, the amount of mathematical discussion is reduced to necessary points to make the idea accessible to a larger audience. The main idea is that many aspects of paper folding can be regularized as patterns of spatial transformation.

Paper folding also found use as a spatial and structural tool in architecture. Paper folding was used as a teaching tool in Froebel taught schools. For architects and designers trained in the Bauhaus tradition, paper folding is fundamental. Moholy-Nagy, one of the teachers at the famous

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[39] Clifford A. Pickover, *The Math Book: From Pythagoras to the 57th Dimension, 250 Milestones in the History of Mathematics* (New York: Sterling Publishing Company, 2009), 310.
[40] Benoit B. Mandelbrot, *The Fractal Geometry of Nature* (New York: W.H. Freeman, 1983).
[41] Britney Gallivan, "How to Fold Paper in Half Twelve Times: An Impossible Challenge," *The Historical Society of Pomona Valley*, Pomona, CA, 2005.

school in Dessau wrote: Another type of exercise…is the manipulation of flat sheet of [paper] into three dimensional structures.[42] Moholy-Nagy arrived at a definition of space more or less analogous to Leibniz' ideas. He identified architecture with space, a reality that can be grasped by sensory experience. He took as point of departure a physical law: 'Space is the relation between the position of bodies.'[43] Although the initial idea by Moholy-Nagy appeared to be relational to scientific ideas of space, he, like other architects of his time, took departure and applied it as an aesthetic perception of architecture. However, one diagram from Moholy-Nagy is particularly important for this thesis. In Figure 2.6[44] he draws out a sequence of spatial experience which occurs along time as a series of interrelated dynamic spirals.

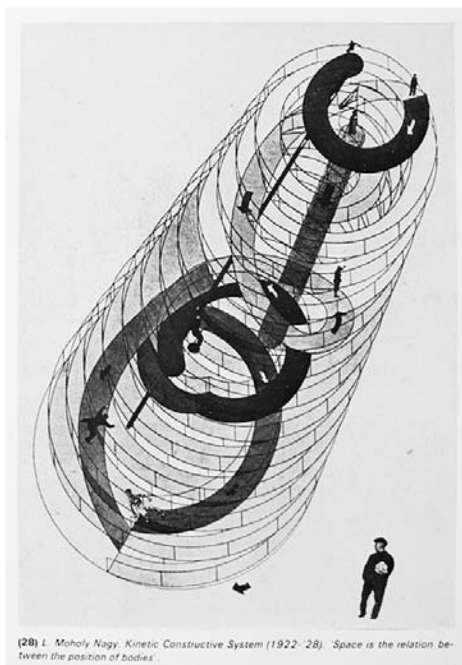

Figure 2.6. Kinetic Constructive System[45]
The image shows Moholy-Nagy's concept of the structure of space
as one perceptually moves through it.

[42]Samuel Randlett, *The Art of Origami,* (New York: E.P. Dutton & Co., 1961).
[43] Cornelius Van de Ven, *Space in Architecture* (Assen: Van Gorcum, 1977).
[44]Samuel Randlett, *The Art of Origami,* (New York: E.P. Dutton & Co., 1961).
[45] Van de Ven, *Space in Architecture, (*1977).

The physical science of space and its concept used in architecture was a tenuous relationship at best. Two well-known architects, Sigfried Giedeon and Charles-Édouard Jeanneret (Le Corbusier) were contemporaries with Einstein, and although they experienced a handful of encounters, never found a way to resolve any relationship between the two fields. This expresses a primary difficulty which architecture has had in resolving its issues with those of science, as it primarily has to do with approach. Einstein saw space-time as a purely mechanical event. Giedeon and misinterpreted the application of ideas such as space-time to architectural ideas of materiality, aesthetic, and spatial experience. As a pun, the architect *generalized* space-time as a blanket theory, where in actuality its application is put in the context of very specific situations.

In La Città Nuova, Sanford Kwinter continues: "In aesthetics, no less than physics, the last years of the nineteenth century and the first of the twentieth brought about a decisive transformation in the concept of space. Beginning with Hildebrand's *Problem of Form* (1893) in which space appears for the first time both as an autonomous aesthetic concept and, more importantly, as a continuum unbroken and indistinct from solid objects," space emerged with a new positivity as an object of both knowledge and direct experience."[46]Although architecture could be classified into urban designs and socio-cultural styles, according to their form in space, space was still relatively little understood in terms of its operative behavior. As a result, architectural theories on space went through a philosophical series of quasi-metaphysical states during the first half of the 20th century. The first modern philosophical conception of architectural space is introduced in the *Problem of Form*[47] by Adolf von Hildebrand (1847-1921) in 1893. In Peter Collins in *The Changing Ideals of Modern Architecture*, the concept of ‘space’ is a relatively recent development in architectural literature: “…It is a curious fact that until the eighteenth century no architectural

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[46] Sanford Kwinter, *La Città Nuova: Modernity and Continuity,* ed. K. Michael Hayes. *Architecture Theory Since 1968* (Colombia University and the Massachusetts Institute of Technology, MIT Press: 1998), 588-596.
[47] Adolf Hildebrand, *Problem of Form* (Strassburg: Heitz & Mundel, 1918).

treatise ever used the word (space), whilst the idea of space as a primary quality of architectural composition was not fully developed until the last few years….(to Classical theorists the notion of space) had no three-dimensional significance whatsoever….its introduction into the history of architectural ideas derives almost entirely from its use by German theorists at this time."[48] The modern interpretation of architectural space follows that conceptual line.

The physical properties space is still a less understood concept in architecture. From *Space in Architecture* architect Cornelius Van de Ven makes a poignant remark: "At this moment all our scientific knowledge of the character of space still comes from explorations on the large scale of cosmology or on the small scale of microphysics. These extreme fringes will always establish a lack of connection with the social scale in which the architect has to operate: the scale of human habitation. It must be clear that inspiration from science to aesthetics of space, which work each of them with distinct levels of scale....Since the aesthetics of space in architecture are in one way or another transcendental expressions of scientific space concepts, it is of great importance to know which expressions of space concepts exist in scientific philosophy."[49] The shift prompted a deeper view about architecture and its possible principles as an interrelated framework of social behavior and configuration. Spatial analyses, such as space syntax suggests that space has an internalized structure which can be described through geometric principles. Space Syntax analysis is a systematic method of architecture describing spatial use developed in the late 1970's by Bill Hillier and Julienne Hanson. Space syntax is based on 8 basic open and closed spatial configurations of buildings and how those combinations affect their social use.

[48] Peter Collins, *Changing Ideal in Modern Architecture* (Canada: McGill-Queen's University Press, 1998).
[49] Cornelius Van de Ven, *Space in Architecture* (Assen: Van Gorcum, 1977).

SPACE GRAPH

SYSTEMS

ch3

3. SYSTEMS: SPACE SYNTAX

> When two things occur together it is easy to assume they share a causative relationship. (Anonymous)

> It is far from obvious that space is, in some important sense, an objective property of buildings, describable independently of the building as a physical thing. Most of our common notions of space do not deal with space as an entity in itself but tie it in some way to entities that are not space. For example, even amongst those with a interest in the field, the idea of 'space' will usually be transcribed as the 'use of space', the 'perception of space', the 'production of space' or as 'concepts of space'. (Bill Hillier, Space is the Machine)[50]

The primary aim of this chapter is to explore how space, as a physical entity can be studied through the regularized patterns found in paper folding. Essentially this chapter explores how spaces are linked with each other in 2-Dimensions. The concept of 2-Dimensions is defined for this thesis according to Cyril Stanley Smith in *Speculations on Dimensionality, Valence and Aggregation.* A 2-Dimensional projection of connected vertices preserves the connectivity of a system of any dimensionality; the world seems to be three dimensional because in three or more dimensions any two vertices can be directly interconnected regardless of intervening structure…everything takes meaning only by interaction with something else…the real units of the world are not particles but connections, and a connection must be between two things; dimensionality in the limited sense used

[50] Bill Hillier, *Space is the Machine* (London: University College London, 2007).

here can be determined by examining the connectivity of vertices forming the network of closure that defines a “thing”.[51]

The first fold, or index fold, not only determines any subsequent folds but also create two distinct, but related spaces in the paper. Figure 3.1 is a set of ‘turned up part’ (TUP) folds. TUP folds are simply single fold paper folds. TUP folding describes the multitude of possibilities which a folding pattern might begin. TUP folds have been explored and regularized by researchers such Kazuo Haga. Haga also determined that the single TUP fold spatial patterns could be mapped out with Set theory through Venn diagrams similar to Figure 3.2 below. In the diagram area A and area B are related by the space of AΠB. For an in depth explanation of TUP folding, please refer to *Origami 3: Origamics.*[52]

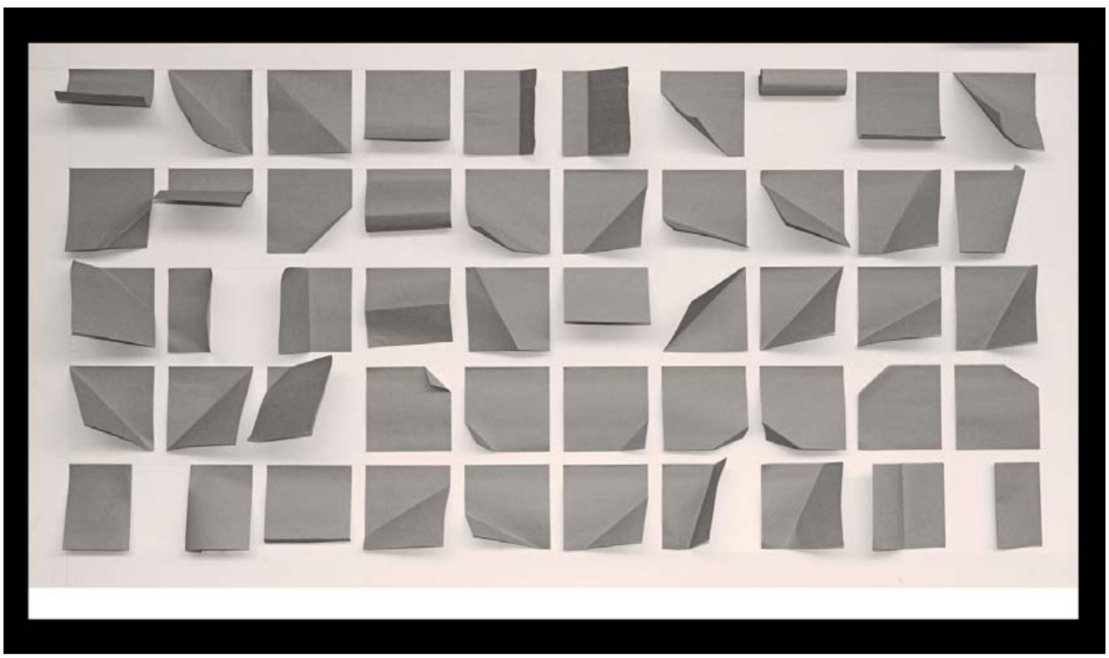

Figure 3.1. Single TUP folds 3"x3" Post-It Notes[53]
TUP folds are virtually limitless, 49 examples are shown. The single example of a
Rolled form is not a TUP. The two sides adjacent to each fold share a relationship
That can be described through a series of ratios and spatial distributions.

[51] David W. Brisson, *Hypergraphics: Visualizing Complex Relationships in Art, Science and Technology* (Boulder: Westview Press, 1978.), 23-33.
[52] Thomas Hull, *Origami 3* (Massachusetts: A.K. Peters, 2002), 307-328.
[53] Folds and photo by author.

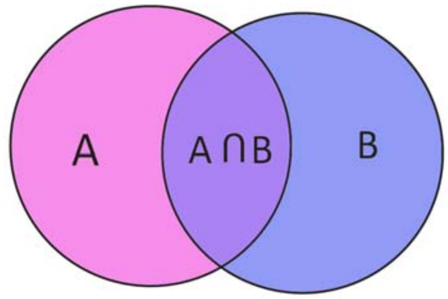

Figure 3.2. Venn diagram, intersection of A and B for a simple TUP fold[54]
This is a diagram of a single sheet of paper representing the relationship
between two sides of a single crease.

Chapter 3 is a deeper exploration of space and paper folding in relationship to Set theory, specifically space syntax. Space syntax used to study the spatial built environment where its theoretical basis is used for comparison in this thesis. Space syntax is an applied means for studying social movement in the built environment. The main difference between paper folding and space syntax is that the methods used in space syntax are based on interpretive features of space rather than a study space directly. That specific issue along with its implications is discussed in Chapter 4. Space syntax is an interpretation of set theory. Set theory is a way to abstractly describe how objects or features of the physical environment are organized spatially, and are used for both paper folding and space syntax analysis. Graphs are mathematical structures used to model pairwise relations between objects from a certain collection; a "graph" in this context is a collection of "vertices" or "nodes" and a collection of edges that connect pairs of vertices. In contrast, set theory is the branch of mathematics that studies sets, which are collections of objects; …any type of object can be collected into a set; …The language of set theory can be used in the definitions of nearly all mathematical objects.[55] Figure 3.3. is a Venn diagram used to describe the interaction seven basic types of architectural spaces in space syntax analysis. The primary goal of space syntax is to

[54] Diagram by author.
[55] Philip Johnson, *A History of Set Theory* (Prindle: Weber & Schmid, 1972).

realistically, and accurately, model the interaction of pedestrian behavior with the spatial structure of the environment, and determine which configurations lead to successful design. The primary data used to correlate space syntax maps are human movement patterns and statistics. At its core, the data is based on how people move according to an environments' visibility of access in conjunction with its physical form.

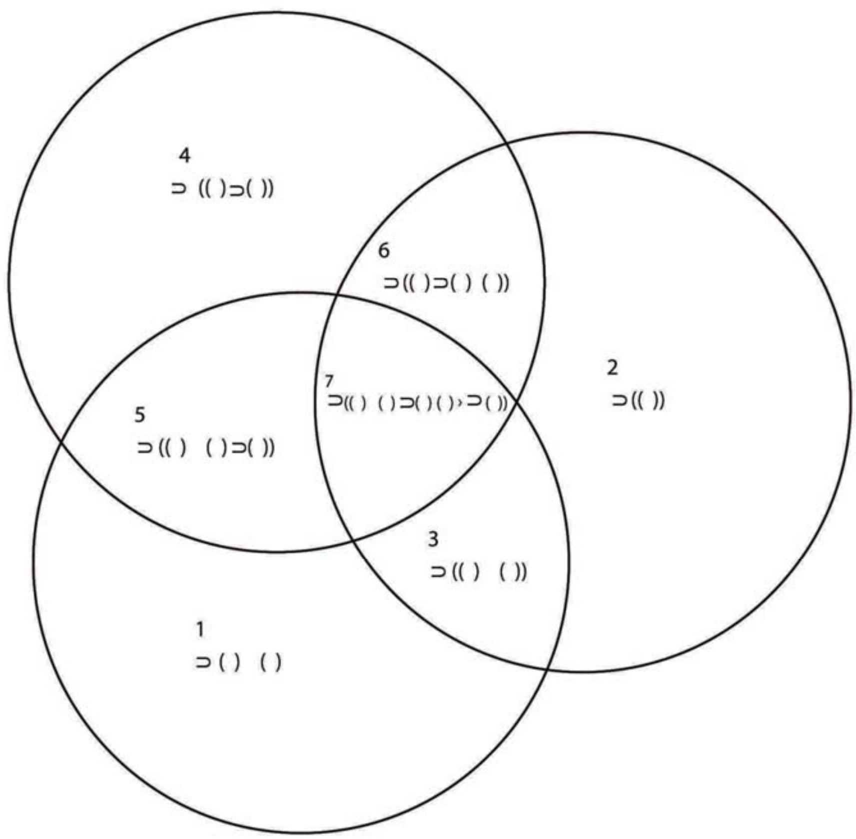

Figure 3.3. Space Syntax Venn Diagram[56]
The diagram shows three basic spaces which overlap to create 7 distinct,
yet interrelated forms. These configurations are the basis for spatial relationships
of building used in space syntax analysis.

[56] Graph redrawn by author. Original source: Bill Hillier, *Space Syntax* (London: University College, 1976), 162.

A flat folded sheet of paper is technically not 2-Dimensional. However, it can be interpreted to function as one. This thesis uses flat paper folding as the main case study. Flat folding behavior is also well understood. According to Jonathan Schneider in *Flat-Foldability of Origami Crease Patterns* flat folding paper folding can both be interpreted from its physical form to an abstract one through geometry without any loss of information as long as it follows a set of folding rules.[57] This is further discussed in Chapter 4.

Figure 3.4 is a diagram of the profile view of a sheet of paper. The labeling indicates three things: 1) an alphabetic designation for the different folds, 2) a numerical description for the point where the folds are made, and 3) a parallel alphabetic labeling which describes the reverse phase of the line or paper. The line can be seen as both physically measurable space and as charges, like an energy structure. The sheet is shown initially as a 1-Dimensional line in level 1. At level 2 a single fold is made which changes the state of the line from 1-D to 2-D. At level 3 and 4 two additional folds are made creating a complex system where two sides B and C touch. When the line is unfolded in level 5 it can be divided into four distinct parts according to the points of folding. The information of the transformation from 1-D to 2-D is 'stored' in the folding points rather than the folded sections themselves because the creases act as the memory of the paper. Although it seems obvious, it is often easy to miss the importance of space through such a simple transformation. This type of folding called 'linkages' has been studied in depth by Erik Demaine, probably the most versed in mathematical paper folding world-wide. In *Folding and Unfolding* (2001), Erik Demaine, proves all the possible configurations in transforming polygon analysis of cycle linkages of one-dimensional objects folded in two dimensions, how the objects can form and completely revert back to its original form, and that the configuration space for all possibilities are linked.[58]

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[57] Jonathan Schneider, "Flat-Foldability of Origami Crease Patterns" (Swathmore College, December 10, 2004).
[58] Demaine, Erik, "*Folding and Unfolding*" (PhD diss., University of Waterloo, Ontario, Canada: 2001).

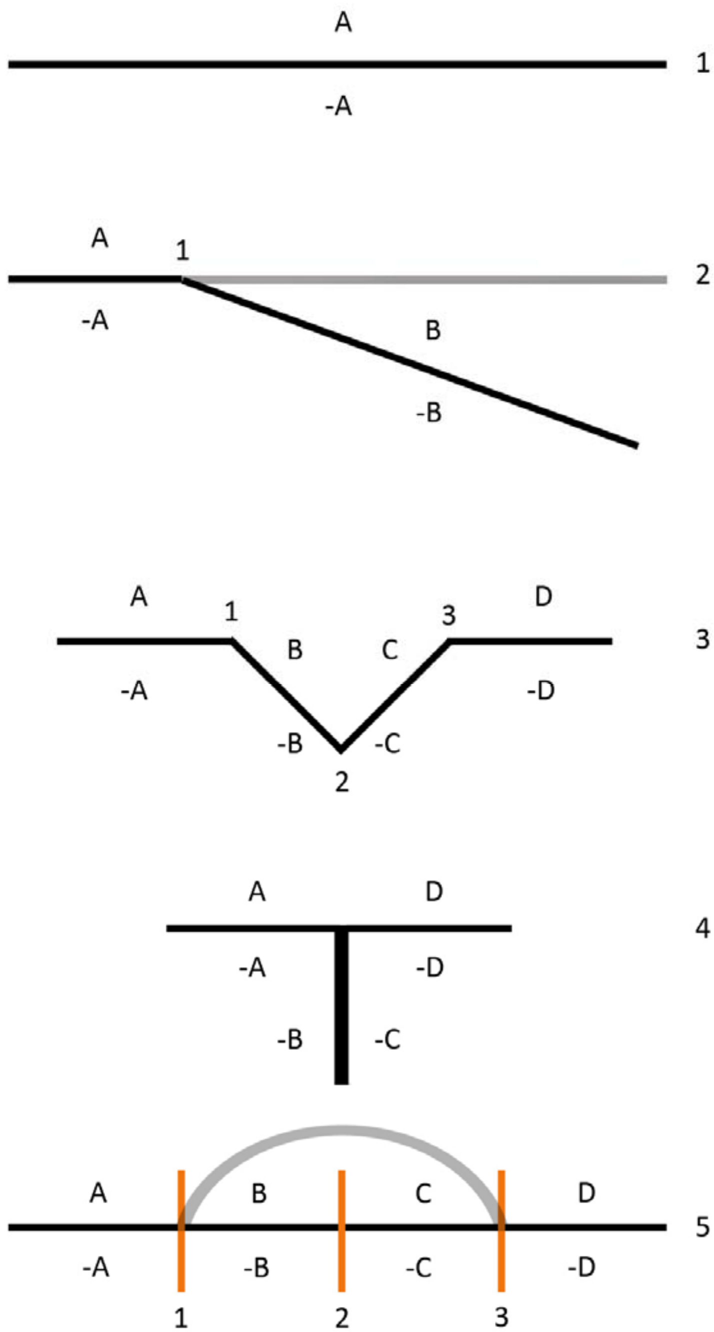

Figure 3.4. 1-D and 2-D Folding Diagram[59]
The diagram shows the profile of the paper initially in 1-Dimensional form.
As the paper is folded it takes on additional characteristics and information.

[59] Diagram courtesy of author.

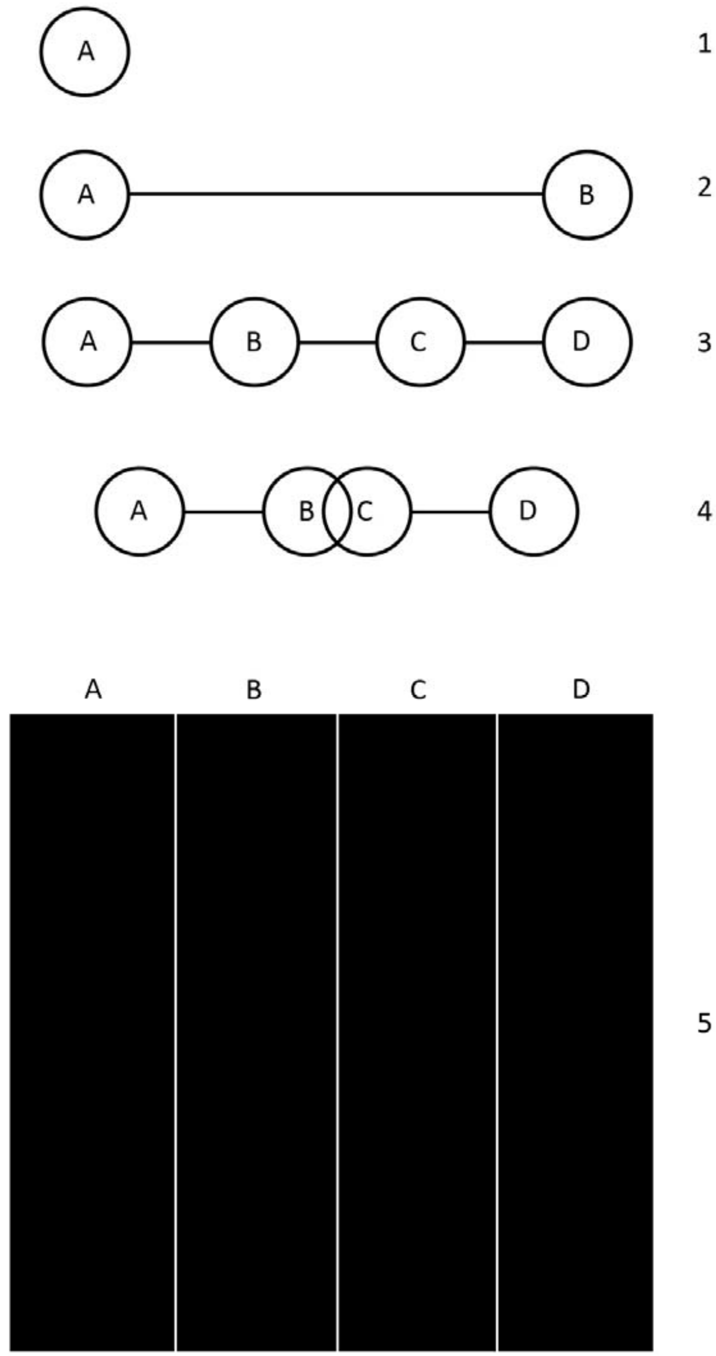

Figure 3.5. 2-D Folding and Tree Diagrams[60]
The upper 4 images show are translations of Figure 3.1.4 into
graph format. The lower image shows the paper in overhead view.

[60] Diagram courtesy of author.

In Figure 3.5 Folding and Tree Diagrams the sheet of paper is looked at in top view in level 5. There are four distinct areas which are created during the folding process. The four area can be interpreted as a spatial tree connecting the areas of the paper as shown in levels 1-4. Levels 1-4 is a parallel description of the folds from levels 1-4 of Figure 3.4. Figure 3.6 makes a distinction in the tree diagram. In the previous diagram the areas or spaces are separated by circles and line. However, one of the conditions for this thesis is that the material environment (paper) and space must be considered as both interdependent and continuous. It is through paper folding that both are simultaneously present in a unique way. An interpretation of what A and B as a fold is shown above level 4 in the diagram. A typical spatial tree diagram of rooms of a building is shown below in Figure 3.6. The circle 1 denotes the entry of the space. The amount of connections describes the 'depth' required to reach the innermost space. The depth of the interrelated space can then be described through a series of ratios.

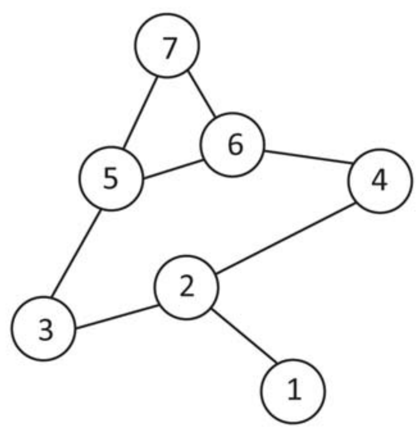

Figure 3.6. Space syntax justified graph tree diagram[61]
The diagram show the depth of any particular space and its relationship to the others.
Node 1 represents the entry, while node 7 is the deepest and has the least integration,
In terms of its integrated distance to the other nodes.

[61] Diagram courtesy of author.

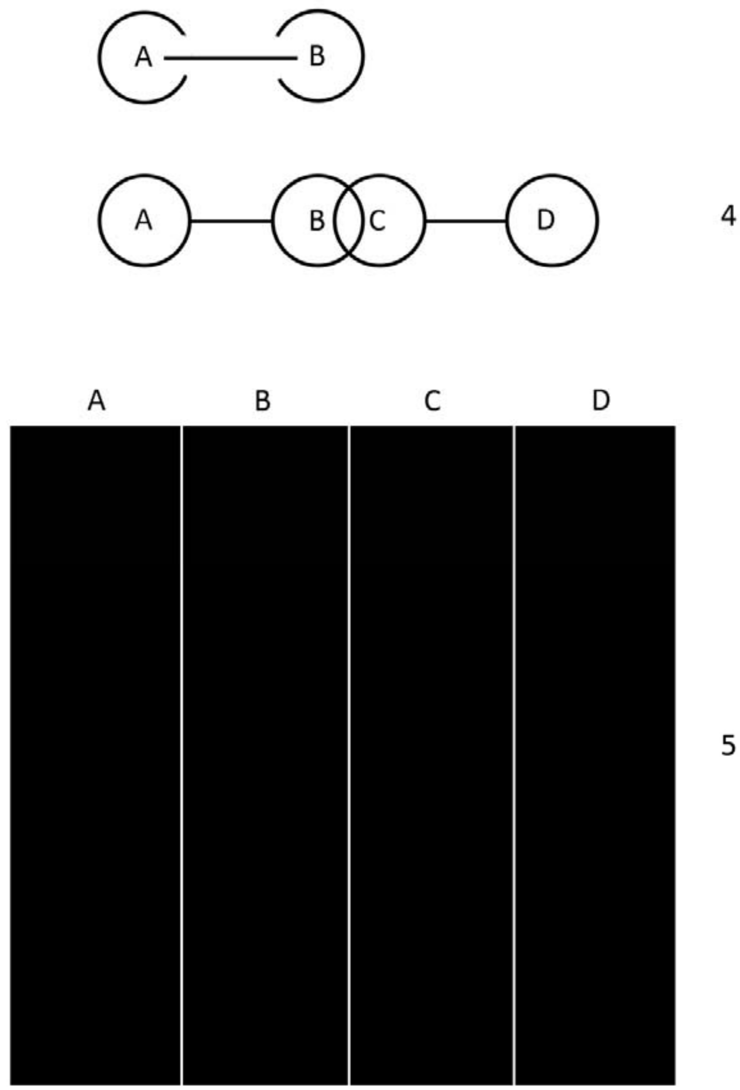

Figure 3.7. 2-D Folding and Tree Diagrams[62]
The lower image shows the division of paper into 4 equal parts.
The upper two images show the unit relationship and the areas
B and C where they are folded.

In Chapter 5 the sheet of paper is placed on a concyclic diagram, which is essentially a circumscribed circle around a polygon. The concyclic diagram allows the vertex of the paper fold to

[62] Diagram courtesy of author.

be plotted as geometric points along the circle, essentially creating another way to visualize spatial information. In level 3 of the diagram is a projection of the folded form as a perspective drawing. The vanishing point is actually along the inside of the circle of the diagram. Although the image is 3-Dimensional in representation the information is maintained even as it is transformed into a 2-D flat fold, but the information is 'nested' in the folds.

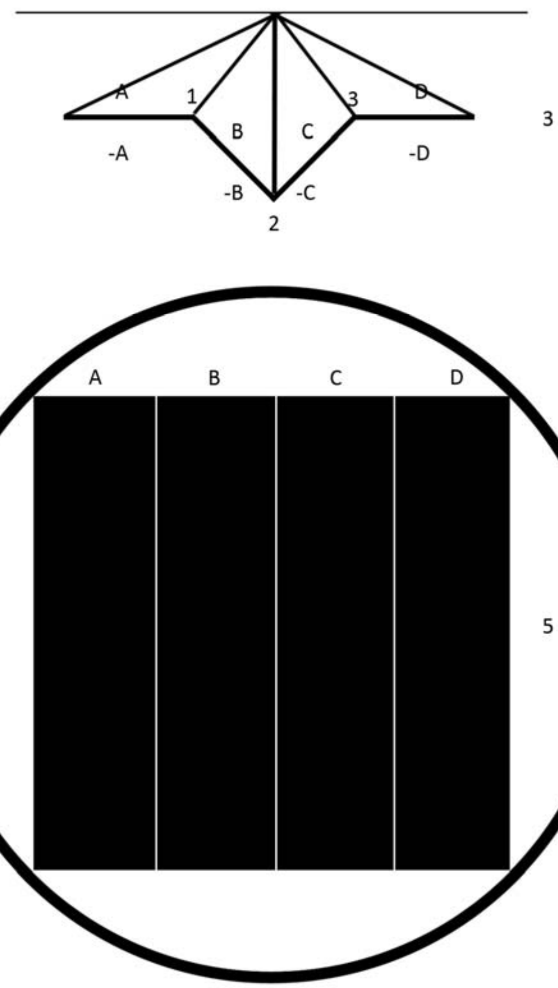

Figure 3.8. Projection and Folding Diagrams[63]
The upper image shows a projection of the folding process from inside the concyclic diagram. The lower image shows how the square sheet is situated with the concyclic diagram.

[63] Diagram courtesy of author.

Although paper folding and space syntax are linked through the concept of space, this thesis is strictly more of an exploration about the operative behavior of space, namely that space can be described as an objective, structural feature of the environment, rather than treating it as a given phenomenon. The premise is that paper folding could serve a much more powerful, and broad role, if it is used to understand more basic operations of space. In the comparison, strong commonalities are found between the main methods of analysis in space syntax which include Depth Map, Isovist Space, Convex Space, and Axial Line Analyses. Although this thesis is not about adapting paper folding as a tool strictly within the confines of space syntax, both subjects share a pattern of regularity rooted in set and graph theories. Paper folding is conceptually simple but its behavior is complex.

The problems of space strongly relate to the issue of the spatial configuration of its parts. Space plays a fundamental part of virtually every activity that humans participate, including the built environment. Until the 1970's the idea of space has been difficult to explain in any quantitative way in architecture. This fundamentally changed when space was used in conjunction with physical configurations of objects in the form of space syntax. In 1976, Bill Hillier and a group of several researchers presented a paper on symbolic syntactic objects representing open, closed, and semi-closed configurations of spaces.[64] The theory presented eight basic spatial configurations of the built environment which could be described mainly through 2-Dimensional analysis. Space syntax correlates the configurations of the built environment with social use statistics. It is one of the few areas of study in architecture where direct and objective correlations about how the spatial configurations of the built environment affects society. Paper folding and space syntax can both be (

Paper folding and space syntax are ways to understand space sequentially, geometrically, and through visual gradients. Spatially, space syntax is understood through Depthmaps;

[64] Bill Hillier, A. Leaman, P. Stansall, eds., "Space Syntax" (Unit for Architectural Studies, School of Environmental Studies, Universty College London, London, England, 1976).

geometrically, it is described through Axial Line analysis; and understood as a spatial phenomenon through visual gradients. Depthmaps are based on Graph Theory. Graph theory belongs to the study of topology, which is defined as a branch of mathematics which deals with the properties of spaces as they form connected pieces and have boundaries, independent of their size and shape. In the case of the axial map, the streets are the vertices of a graph, and their interconnections the edges; the various depth, status, and integration measures are well-established topological parameters.[65] Graphs are among the most ubiquitous models to represent both natural and human-made structural organizations. Graph theory generalizes the spatial environment by showing how its parts are related through simplified diagrams. The graphs can be used to model many types of relationships and process dynamics in physical, biological and social systems.[66]

Depthmap is a specific term used by space syntax. In Depthmap analysis spaces are analyzed through a series of links reminiscent of a string of pearls or trees, the pearls represent rooms or buildings and the string signifying the connections between them. The method shows graphically how spaces are either shallow or deep. A shallow relationship signifies that a series of spaces are more accessible by people, such as a public gathering area in Figure 3.9. Item “d” shows two evenly space rooms with an equal value of 2. However, in Item “e” shows that room b has increased in depth and results in a value of 3. Isolation of rooms such as this in more complex examples can show up as more person or even sacred spaces. More elaborate Depthmaps are based on iterations of this simple diagram. The Depthmap provides information as to how spaces are linked, rather than show any apparent scale. As a result, the graphs are normally shown along-side floor plans of a building or urban environment. In terms of Depthmap, the primary difference between the hypothesis for this thesis is that paper folding can be used to account for the information of space itself in terms of the different spatial conditions.

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[65] Carlo Ratti, “Urban Texture and Space Syntax: Some Inconsistencies” (School of Architecture and Planning, Massachusetts Institute of Technology, March 2003).
[66] Berge, Claude, *Théorie des Graphes et Ses Applications* (English edition, New York: Wiley, 1961).

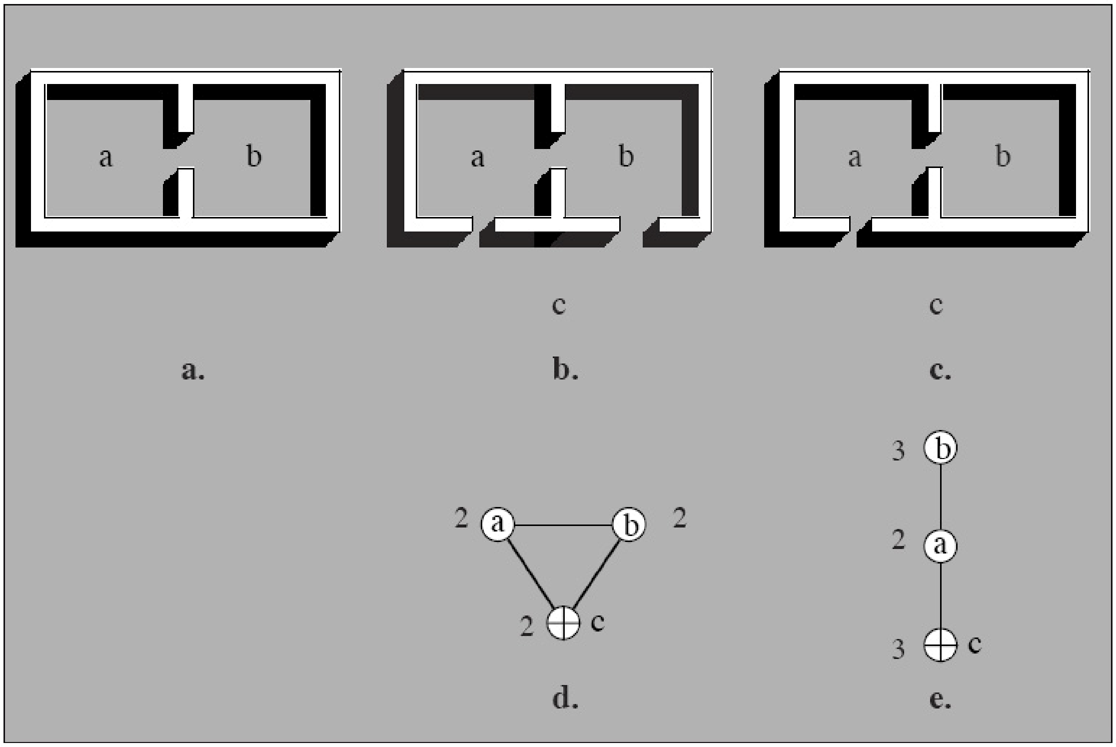

Figure 3.9. Space Syntax Depthmap[67]
Item a does not have a justified graph because it is totally enclosed.
Item b shows an equally divided graph because both spaces are accessed by equal means.
Item c shows that space d is less integrated therefore, having a deeper graph.

A second method used by space syntax is axial line analysis. In *The Space of Innovation*, Penn and Vaughan, 1999, axial lines are important in the structure of space because they link it together linearly as a series of possible movement routes between two points of space.[68] Axial lines can also be converted to X-Y linear graphs. According to Bill Hillier and Alan Penn, *Rejoinder to Carlo Ratti*, axial analysis is strictly about the continuity, or discontinuity, of lines in an urban configuration as confirmed through morphological and behavioral studies.[69] Further, Carlo Ratti also notes that in the urban texture pattern…shows a more general phenomenon: the *binary* nature of topological

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[67] Bill Hillier, *Space is the Machine* (London: University College London, 2007).
[68] A Penn, J Desyllas, and L Vaughan, "The Space of Innovation: Communication and Innovation in the Work Environment" (London: University College London, 1998).
[69] Bill Hillier and Alan Penn, "Rejoinder to Carlo Ratti" (London: University College London, November 2003.

transformations. Informally, a graph is a finite set of dots called vertices (or nodes) connected by links called edges (or arcs).In terms of paper folding, axial line analysis shows a significant similarity with the circle packing method (CRM) found in paper folding in Figures 3.10, 3.11, and 3.12. In paper folding, axial lines form and are integrated in several distinct ways. An axial line is roughly analogous to the map of creases in circle packing. Note that Figures 3.10, 3.11, and 3.12 show CRM diagrams and trees for 3-Dimensional folding. However, the examples such as the folding crane and the ones used for the case studies are flat foldable although they may also take 3-Dimensional form as in Figure 3.10 below.

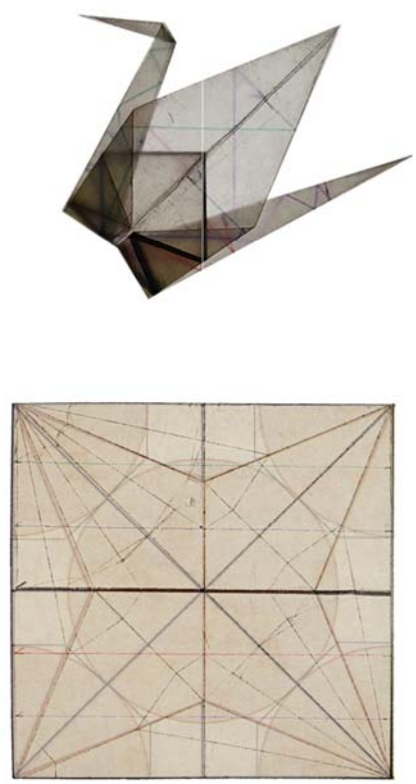

Figure 3.10. Paper Crane Overlay with Circle River Method Fold Diagram[70]
The paper crane crease pattern is overlaid the CRM diagram
from Figure 3.11 to show the layout of the spatial pattern.

[70] Fold and diagram courtesy of author. Underlying image: Robert J. Lang, *Origami Design Secrets* (Massachusetts: A.K. Peters, 2003), 375-408.

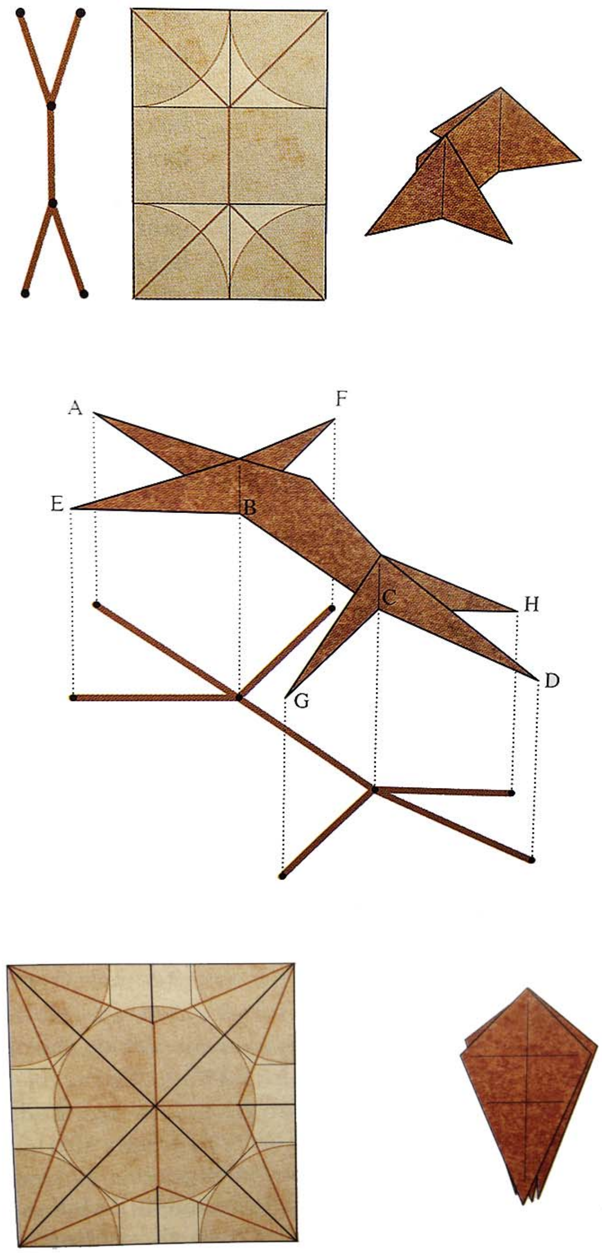

Figure 3.11. Circle River Method Fold with Trees[71]
CRM folding diagrams have a deliberate organization based on spaces which form the parts of a model, the circles, and their crease patterns, lines. Paper folding is unique because it is highly constrained compared with spaces of the built environment such as in Figure 3.13.

[71] Robert J. Lang, *Origami Design Secrets* (Massachusetts: A.K. Peters, 2003), 375-408.

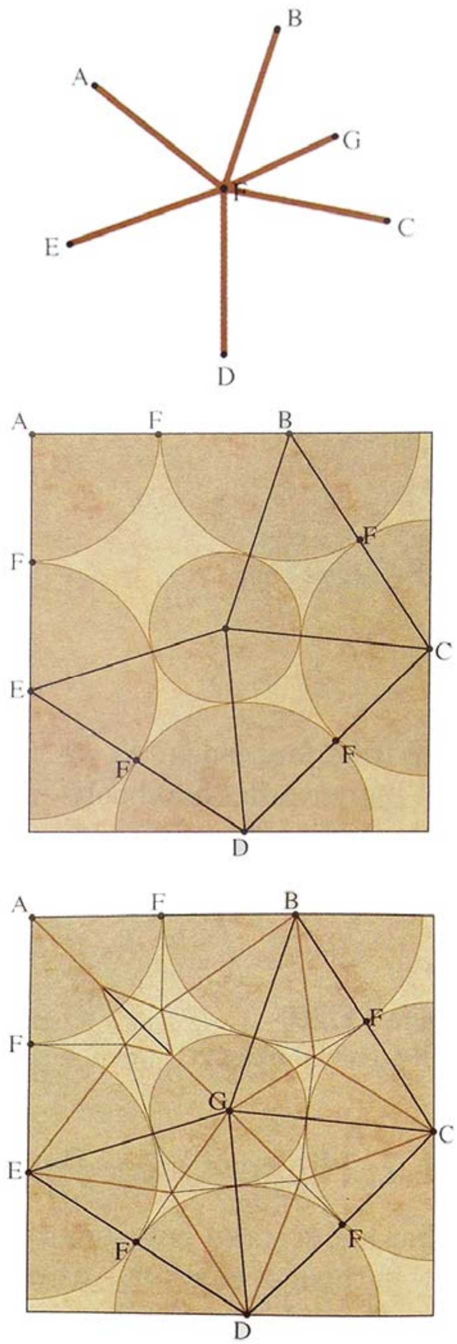

Figure 3.12. Circle River Method Fold[72]
This CRM diagram describes the sequential ordering of the folding sequence. Notice
The point F, although not at the center, forms a central relationship to the other points.

[72] Robert J. Lang, *Origami Design Secrets* (Massachusetts: A.K. Peters, 2003), 375-408.

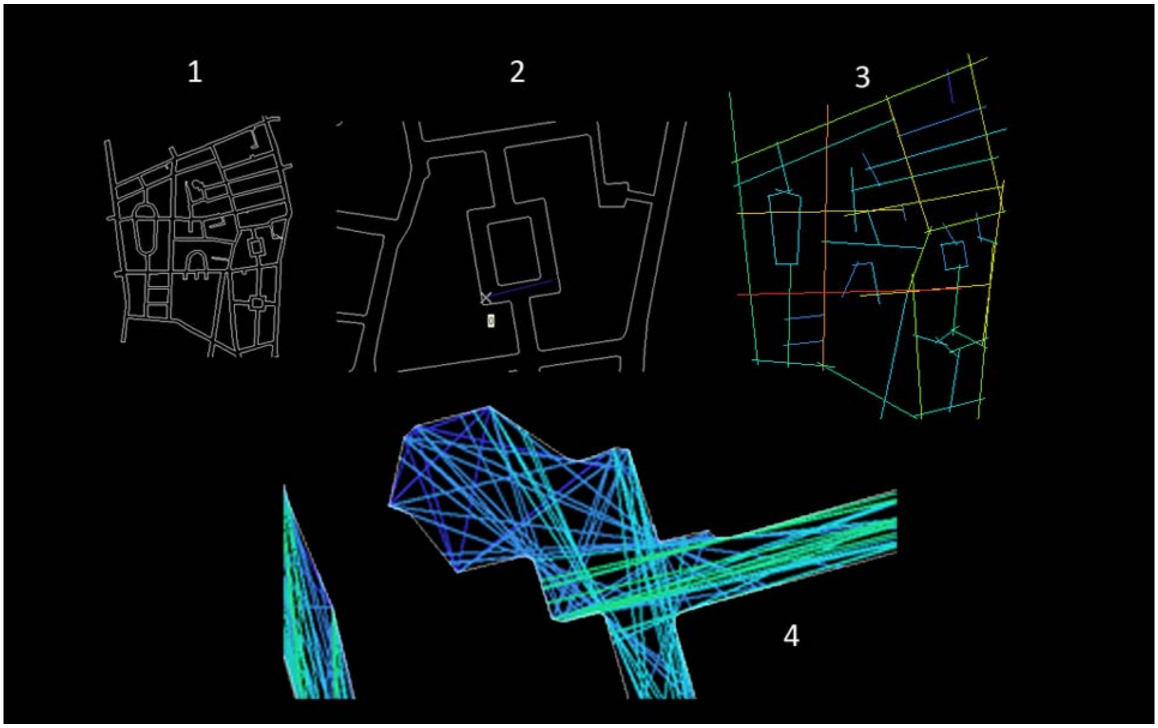

Figure 3.13. Space Syntax Axial Line Analysis[73]
The diagram shows the stages of an axial line analysis. Item 1 shows a chosen street map. Item 2 is a close-up view of a street section. It shows a connection between two spaces, denoted by the "x" and the blue line. Item 3 is a fully developed map, red denotes highest use, while blue the lowest. And Item 4 shows a close-up view shows the complexity which axial lines can take.

Figure 3.13 shows the development of an axial line map. Although at a completely different scale than paper folding the core principle remains the same. In item 1 a map is chosen for the analysis. In item 2 a space is chosen to begin the analysis by connecting one point in space to another, in terms of visibility and access. In item 3 a full map showing the average intensities of axial lines are shown (red areas denote highest use). Item 4 is a close-up section of the map showing the array of line connections from any single point. Axial line analysis works very similar to CRM in that they connect high use spaces to one another by creases of line connections. However, paper folding has a more specific form of organization that is highly sequential.

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[73] Alasdair Turner, "UCL Depthmap 7: Axial Line Analysis: *Version 7.12.00c"* (London: University College London, 2012).

A third method of space syntax is isovist analysis. From Isovists to Visibility Graphs, 2001, Alisdair Turner discusses the idea of isovists. The Isovist or viewshed area was first developed in 1967 by C.R.V. Tandy. The idea was further developed by M.L. Benedikt in 1979 who introduced analytic measurements to achieve quantitative results from space by simplifying the volume of a polyhedron by analyzing it in slices. An isovist is the area in the spatial environment directly visible from a point location within the space, or a set of point locations within space, which can be used to generate a graph, where the built environment is generally a static quantity in Figure 3.14,Point Isovist. The light blue area if the isovist and the dark blue squares the buildings. Isovists operate in plan form, vertical sections, and 3-Dimensionally. Isovists are different from axial line maps because they show volumetric relationships of spatial configurations through ratios of visibility. In traditional paper folding, especially in flat folding, the idea of the amount of visible space is apparent. When any fold is made over another, the action hides a part of the overall paper folding object. The graph also has a center-point which represents a fold's visibility at any point in time of the folding process.

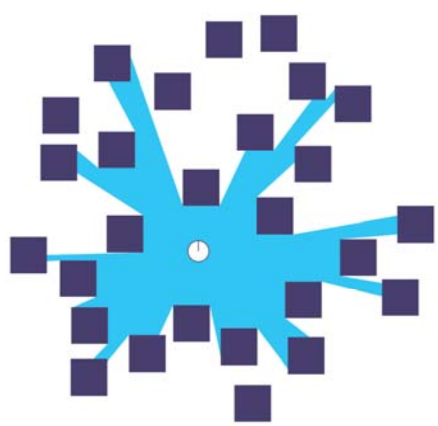

Figure 3.14. Point Isovist[74]
The central viewpoint is the white circle. The dark blue
Squares are buildings. The light blue rays show the amount of visibility.
The relationship are described as a series of ratios in space syntax.

[74] Ben Doherty. 2011.

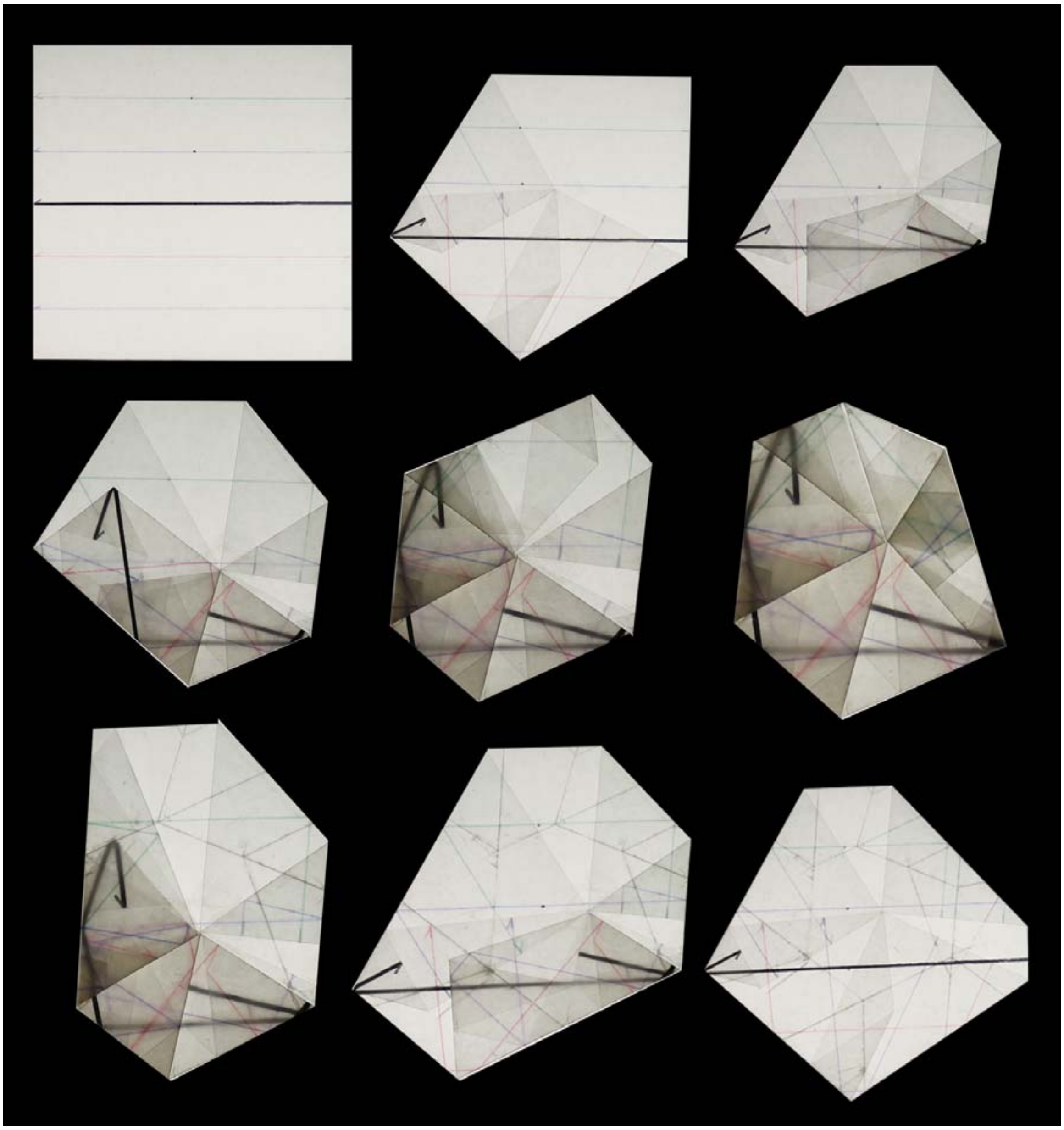

Figure 3.15. Visibility of Layers in Central Fold[75]
The diagram shows samples of a single folding sequence from undifferentiated state, to completed fold, and towards an unfolded state. The folds have isovist properties which change over time.

The paper fold diagram in Figure 3.15 can be directly overlaid with a space syntax isovist simply for the purposes of visualizing the depth of the space in terms of layers. The case studies of folds used for this thesis are all based on a central axis, much like the central viewing point in the

[75] Folds and photo by author.

space syntax isovist. In Figure 3.15 if a viewer stands at the center of the sheet of paper and looked outwards one could visualize that the “density” of any particular area changes with the folding. It is very similar to how much of a space can be seen in a particular configuration. Like isovist space, the layers in paper folding are generally not entirely visible. However, this thesis integrated the use of semi-translucent wax-paper to make the layers and resulting structure visible. Figure 3.15 is a timeline of an irregular central fold pattern. The black and colored lines represent primary and secondary hyperplanes which are further discussed in Chapter 4. The reader will notice can see the distribution of layers according to their visibility gradients. In this sense paper folding is much more sophisticated as a spatial tool, at least in the case of 2-D. The key idea between methods used in space syntax analysis and paper folding is that the information generated in space syntax are primarily point to point relationships. This is a result of cutting ‘space’ which actually leads to a loss of information, simply because an underlying aspect of space is not incorporated. Though it serves the purpose for space syntax analysis, the idea for this thesis to understand space need to take and preserve the information from dimensional changes into account. The next Chapters suggest how this could be done.

SPACE GRAPH

HYPERPLANE

@ch4

4. HYPERPLANE

Thirty spokes converge upon a single hub;

It is on the hole in the center that the use of the car hinges,

We make a vessel from a lump of clay;

It is the empty space within the vessel that makes it useful,

We make doors and windows for a room;

But it is these empty spaces which make the room livable

Thus while the tangible has advantages;

It is the intangible that makes it useful.

Lao-Tzu, c. 550 B.C.E.[76]

Practicing architects may claim that their real expertise is the skill to design space; Thomas A. Markus finds irony in the architects claim that they design space when most of their effort goes into designing the elements that enclose space, into shaping the physiognomy of the surfaces of those elements; despite this, he states, they create space, but rather as a kind of by-product; their practice does not ignore spatial consequences; there must be some kind of knowledge about space, and of course about its functionality; in its tacit form this is a knowledge shared by all of us, by designers and users of space alike.[77]

[76] Laozi, *Dao te Jin.* (Tokyo: Kodansha International, 2010).
[77] Mir Azimzadeh and Bjorn Klarqvist, “Proceedings: 4th International Space Syntax Symposium” (London, 2003).

The most important constraint which this thesis deals with is the issue about how to represent space through paper folding. Ironically, the concept of space is not used as a tool to create volume in the folding process or shapes, but as a means to 'cut' the paper without cutting it. Space is represented as a line, more specifically a mirror or hyperplane, as it is called in geometry. A hyperplane essentially divides a surface into two initial parts. A hyperplane has no dimension and can be considered a continuous entity, much like space. In 2-D form the hyperplane is a line. In its 3-D iteration the hyperplane acts as a plane. A hyperplane can be used in any number of dimensions. However, in this case it will be used in 2-Dimensions. There are some references of hyperplanes being used in paper folding, but this appears to be a unique application.

Several requirements need to be met for the use of the paper medium the hyperplane. The layers need to be visible, though not transparent; the sheet of paper should show the crease-line patterns; and that the paper could be graphically marked for the hyperplane line. To distinguish between the front and back of the paper the hyperplane line is marked on only the facing side of the sheet. Although, two-sidedness is an important and integral feature of paper folding, a two color sided paper was not necessary as the information of the sidedness is taken into account in the cyclic quadrilateral graphs. The mirror line or other graphical features on the paper are made using permanent markers. Wax-paraffin paper is used for the main case studies. Wax paper has several beneficial features. It can be cut to high precision with a blade, it is thin (1/1,000th inch or 0.025 mm), creases are highly visible when placed in front of a contrasting dark-colored surface, and the amount of layers can be reasonably determined according to gradations. The gradations serve as an index for the amount of layers present, the phases (or sides) of the paper, and has properties of Venn diagrams. Wax-paraffin paper has less stiffness than traditional paper used in paper folding. Wax-paraffin is easily subject to wrinkling and damage. Such wrinkling would normally affect the aesthetic qualities of the fold if used for artisanal purposes. However, this thesis focuses on flat folds, which tend to be simpler and have less stress on the medium. The sheets are squares at a modular size of 3x3 inches or 6x6 inches and measured in imperial units. To reduce systematic error and irregularities in the wax-paraffin paper the sheets are outlined using a single template and cut on sheet glass. The folds used in the case studies are illuminated using a Bretford Acculight Light Table, Model: 6000. It uses a fluorescent light source of 5000K, considered for neutral daylight-like lighting.[78] All photos are taken with a 15 mega-pixel DSLR camera and basic studio equipment where necessary.

During the folding process the hyperplane becomes embedded visually in the layers and can be identified according to what side of the sheet is facing the viewer. Figure 4.1 shows the application of the hyperplane to a 6”x6” sheet of wax-paraffin paper. The black line represents the primary hyperplane, whereas the colored lines show secondary ones. The figure shows an unfolded sheet, a basic folded form, and gradient used as a scale for the layer gradients in Figure 4.2. In Chapter 6 the resulting graphs are shown with the folds. The second image in Figure 4.1 shows three basic folds with three versions of the hyperplane. It was decided that a series of 5 lines in different colors proved the most effective for tracing the paths of folding. Figure 4.3 shows several types of basic polygons for reference. There are probably as many paper types as there are possible initial shapes these sheets can take (square, circle, strip, hexagons, and so forth). The variety of possible paper folding types has yet to be formalized, let alone studied for their various technical properties. Since most of the existing literature in paper folding is dedicated to square sheets, this thesis resolves the explanation within that constraint. Figures 4.4, 4.5, and 4.6 are developed folded objects fully using the hyperplanes and the advantages of the wax-paraffin paper. This thesis explores the research problem through the single flat fold process only. Simultaneous and multiple folding processes are technically and computationally outside the scope of this thesis.

[78] B&H Professional Studio Sourcebook.

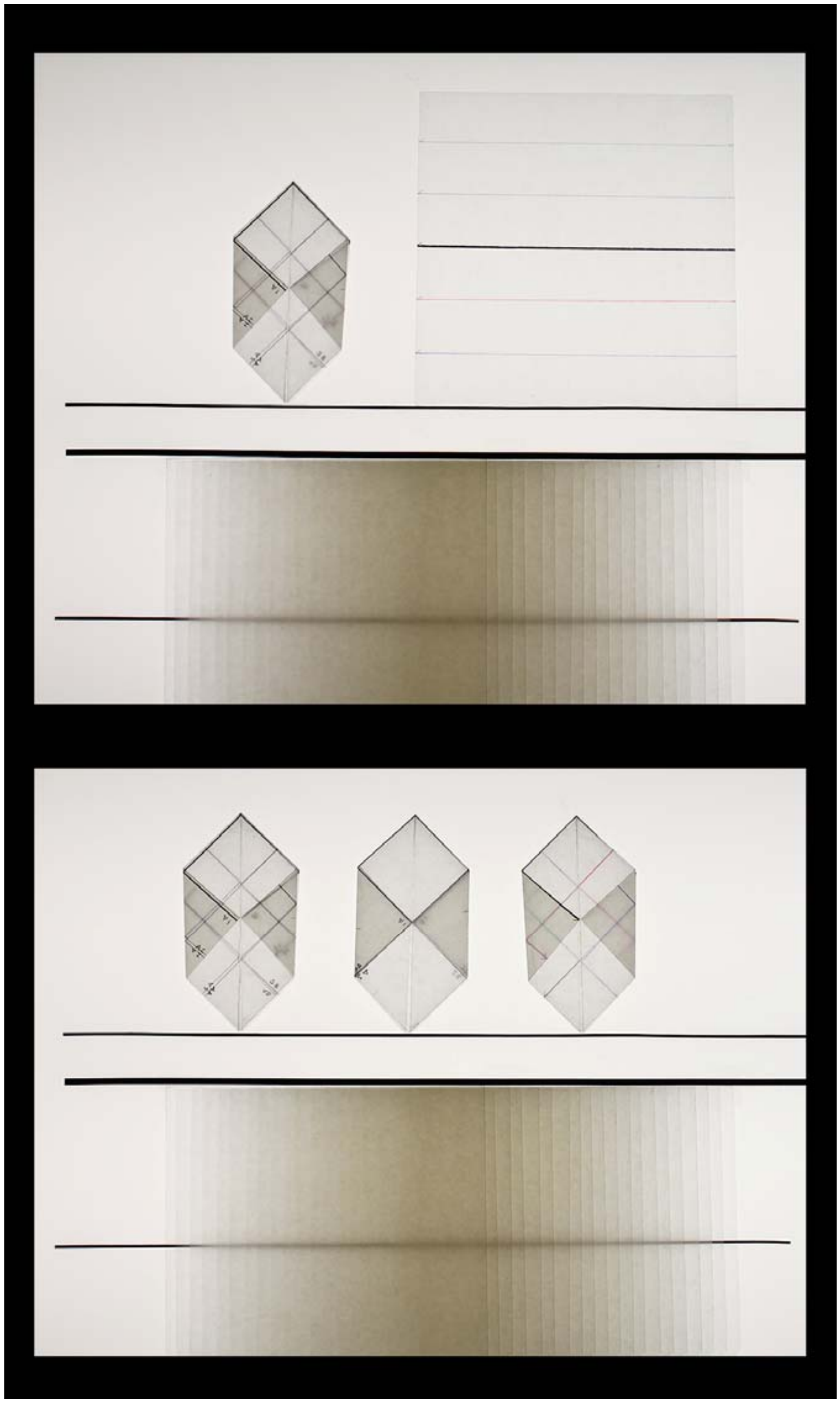

Figure 4.1. Hyperplane, Wax-Paraffin Paper[79]
Several examples were tested to determine the best format for the case studies.
The third diamond shaped fold with colored lines is used for the case studies.

[79] Folds and photo by author.

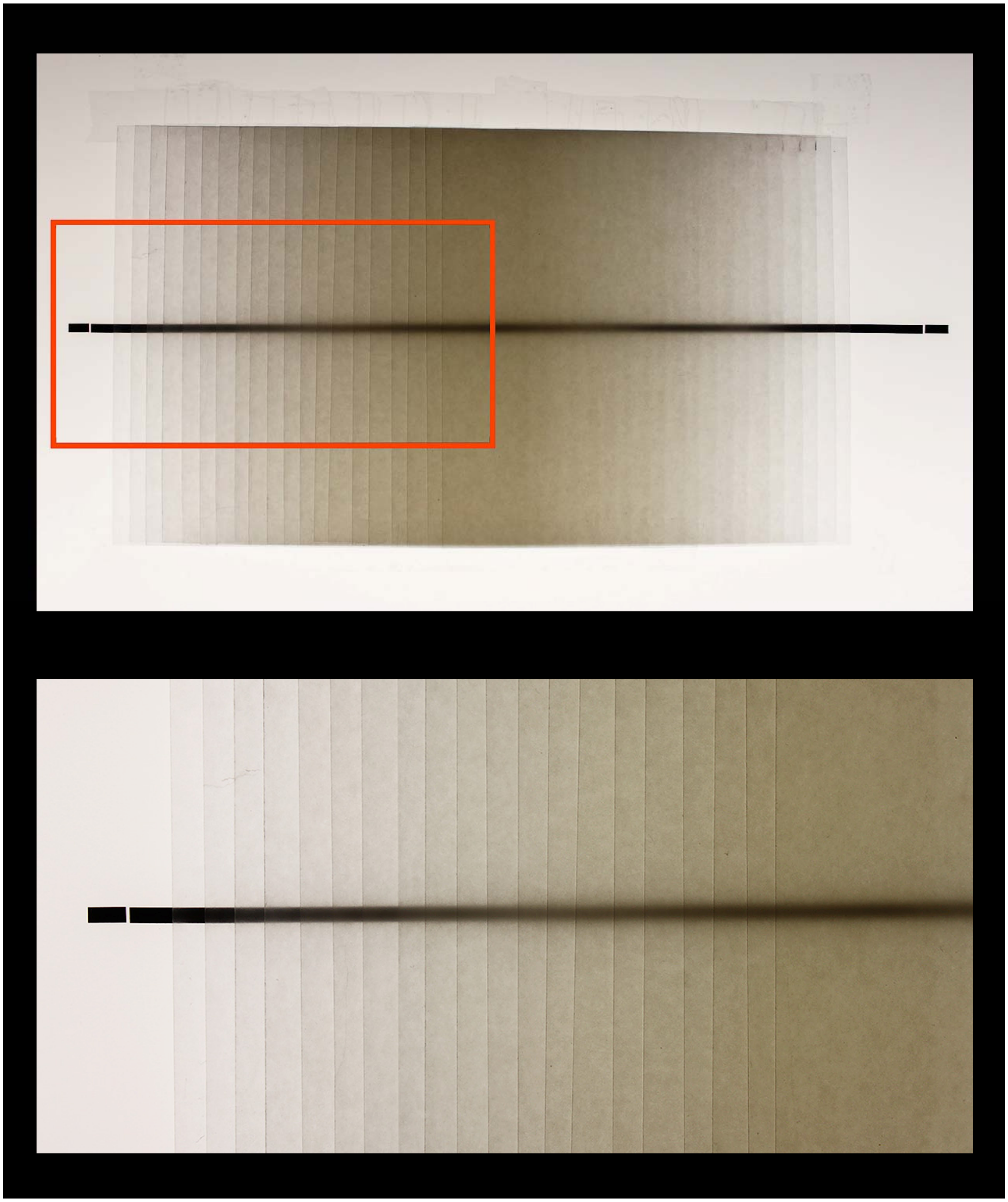

Figure 4.2. Gradient, Wax-Paraffin Paper[80]
The gradient scale is used to compare the average amount of layers in a folded case study and its effect on the hyperplane that is represented by the black line.

[80] Folds and photo by author.

Figure 4.3. Basic Shapes, Wax-Paraffin Paper[81]
Several types of polygons were studied for their effect on folding and gradients.

[81] Folds and photo by author.

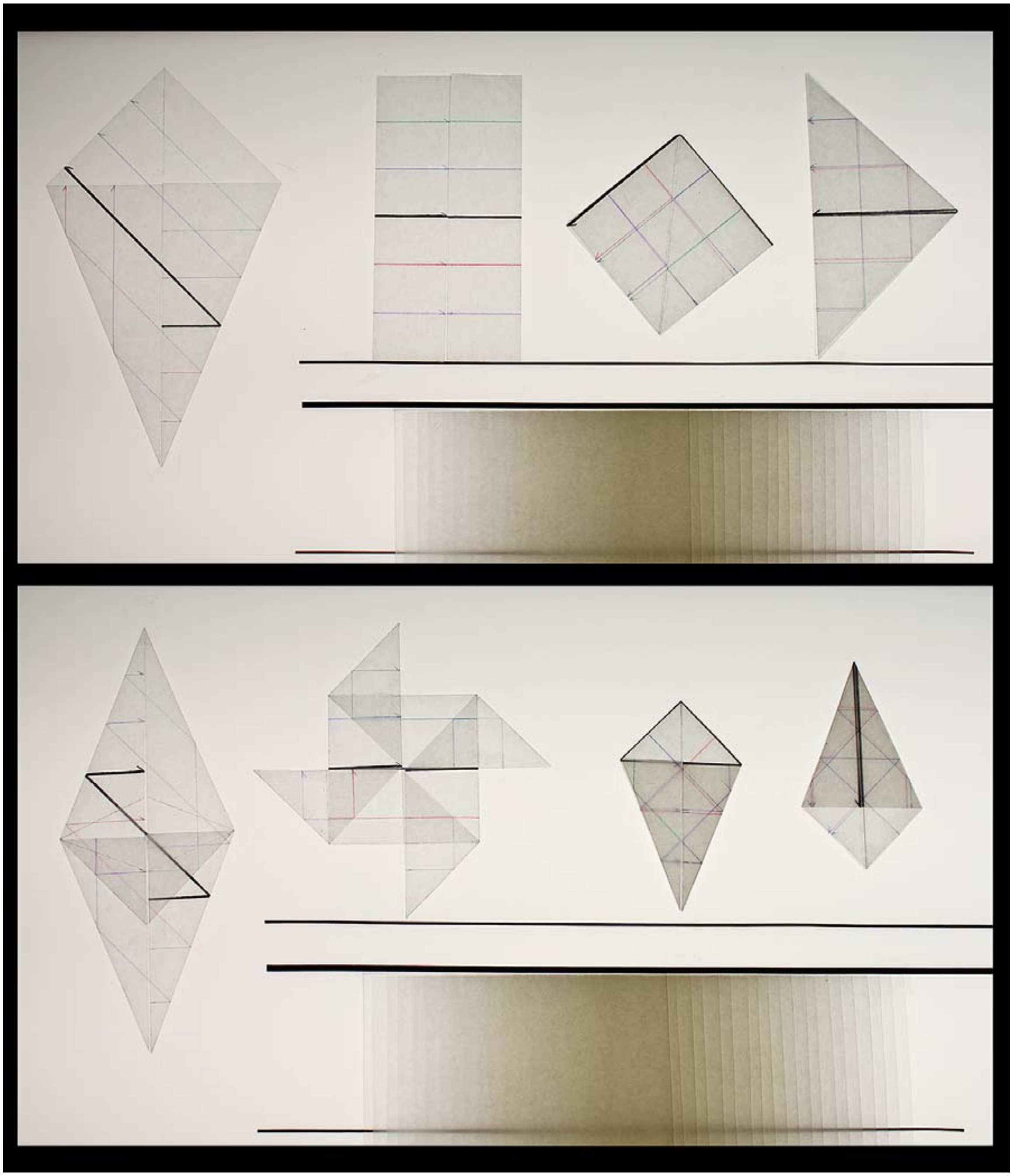

Figure 4.4. 4 Bases, Wax-Paraffin Paper[82]
The top half image shows the 4 basic configurations, called *bases*, used in paper folding.
The lower-half of the image shows the further folding development of the bases.
Notice the effect of the bases on the hyperplane lines.

[82] Folds and photo by author.

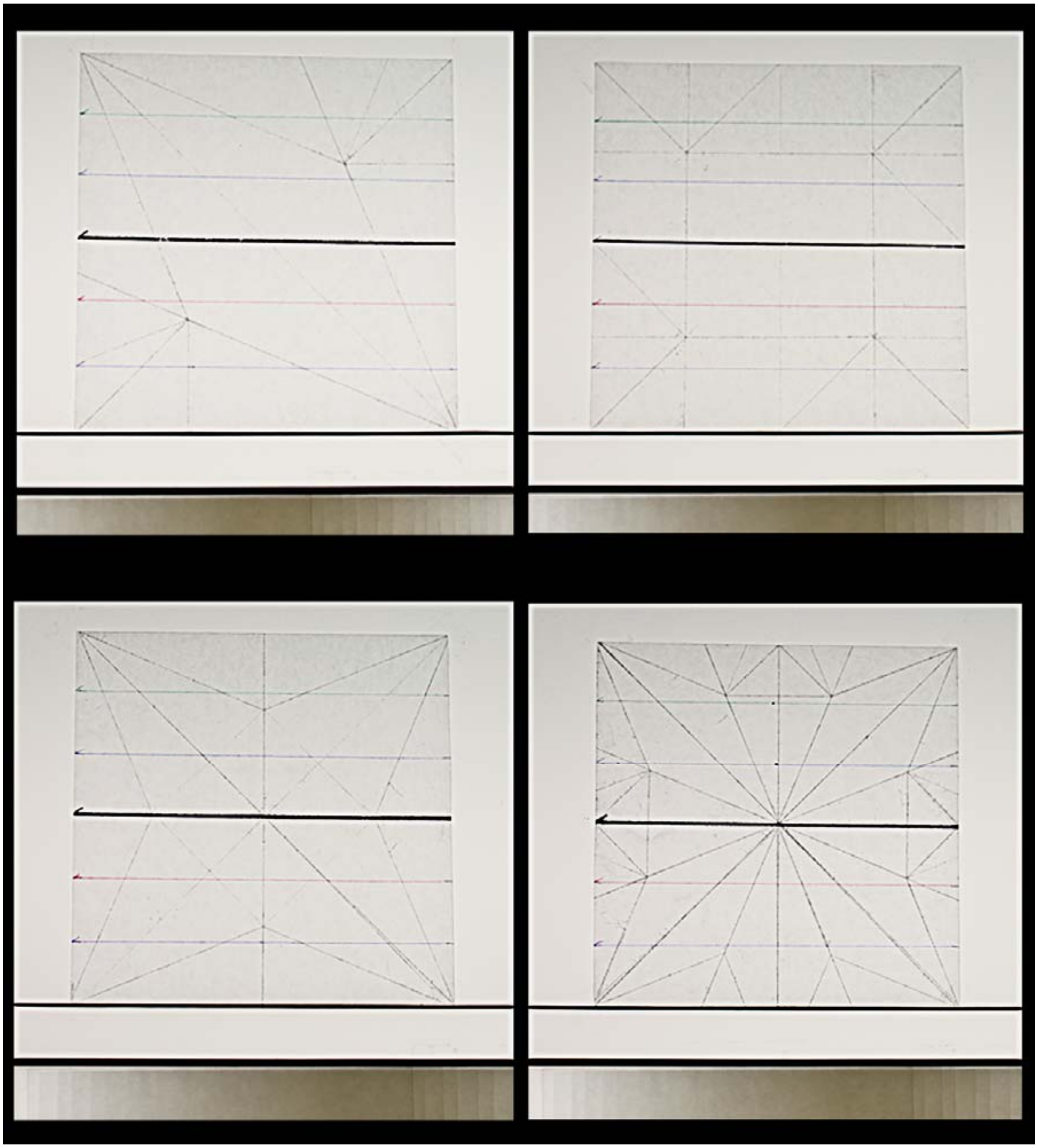

Figure 4.5. 4 Bases, Wax-Paraffin Paper[83]
Above are the unfolded forms of the 4 bases of the lower-half image from Figure 4.1.2.
The top left sheet is the first base, while the lower right image is the fourth base.
Notice that the hyperplanes are segmented by the folding process.

[83] Folds and photo by author.

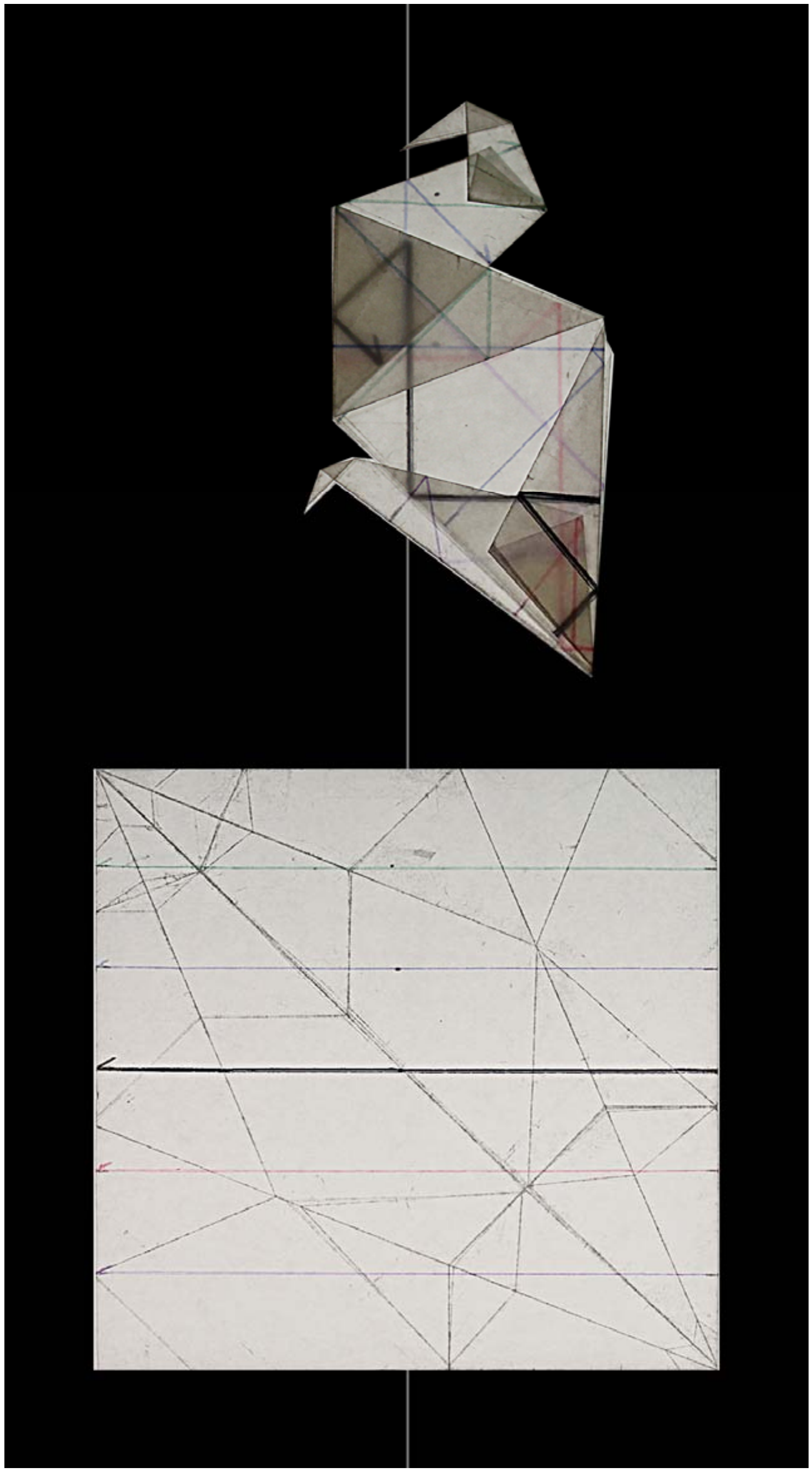

Figure 4.6. Case Study: Animal 1, Wax-Paraffin paper[84]

[84] Folds and photo by author.

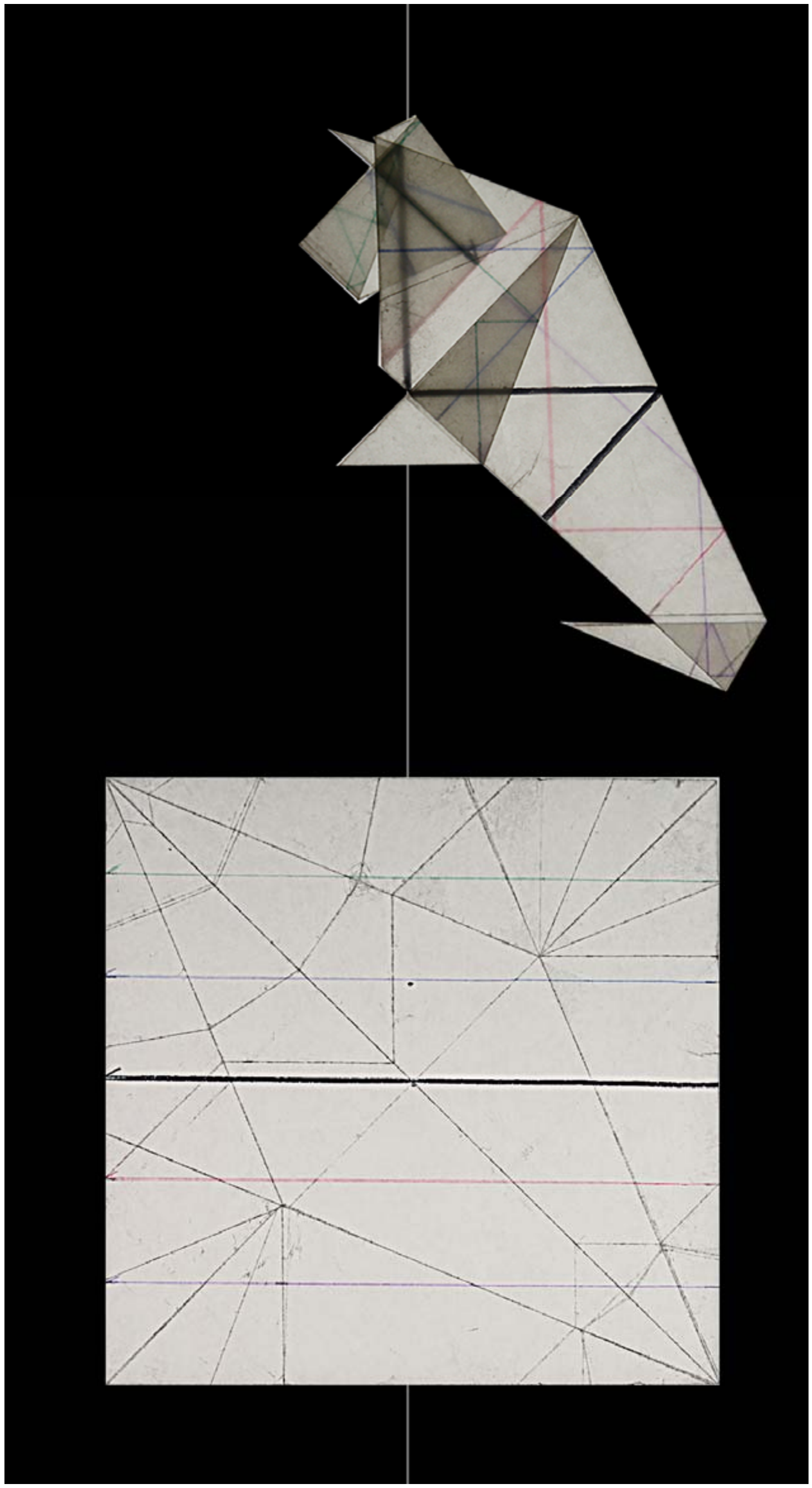

Figure 4.7. Case Study: Animal 2, Wax-Paraffin paper[85]

[85] Folds and photo by author.

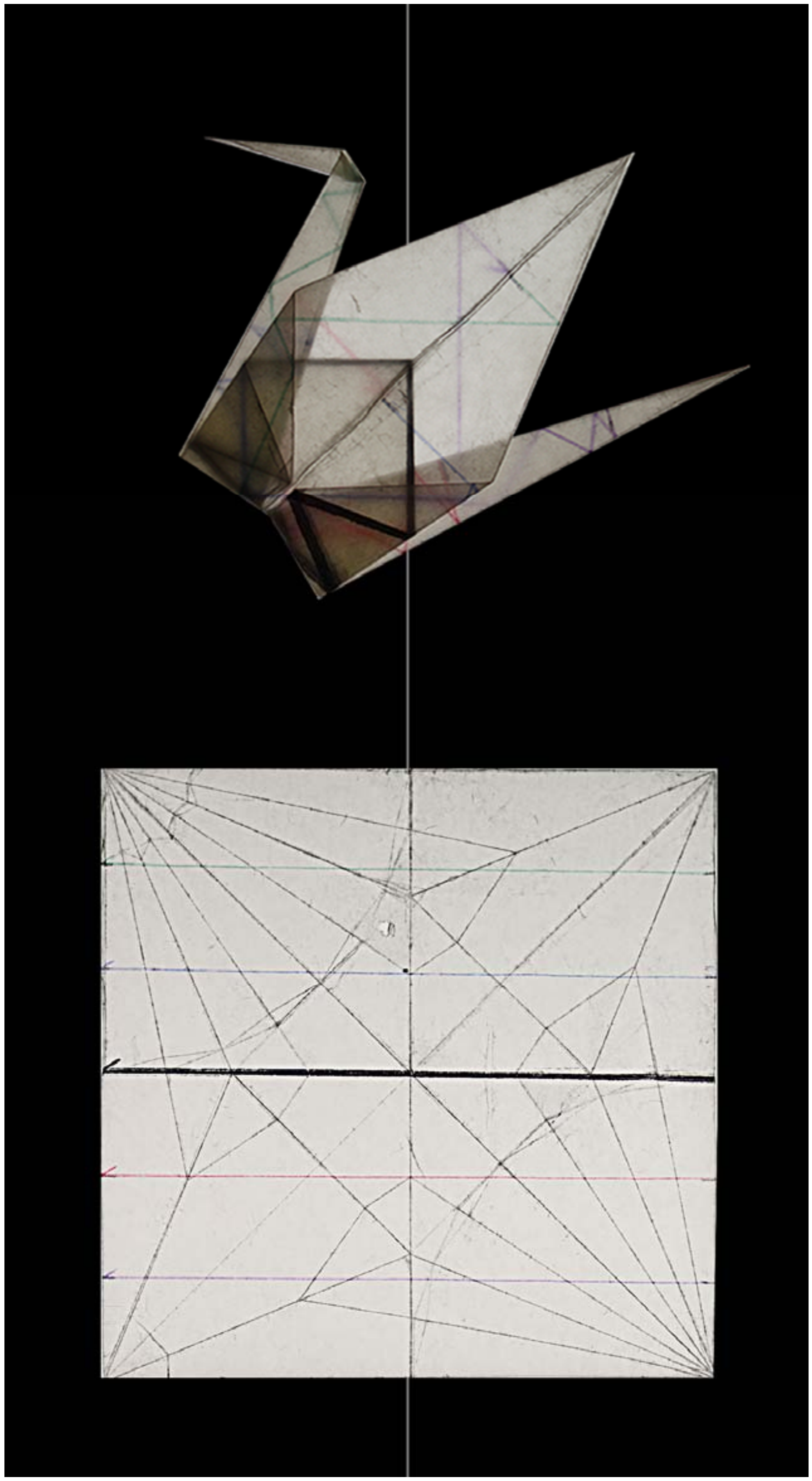

Figure 4.8. Case Study: Animal 3, 6” x 6”, Wax-Paraffin paper[86]

[86] Folds and photo by author.

In addition to the constraints set forth at the introduction, there are several principles for the folding process which this thesis adheres. They include: the four general laws of paper folding stated by Robert J. Lang and the Martin equivalent of Huzita's 06 Axiom.

Four general laws of paper folding[87]:

1. Any crease pattern can be created with two colors (i.e. two-sides).
2. Mountain and valley folds differ by just 2, 2 more or 2 less, nothing more.
3. The angles in a circle, all even add up to 180, and all odd add up to 180 degrees.
4. Layer ordering, stacked, a sheet can never penetrate a fold.

There is but a single axiom shown by George Martin in 1998 which describes all possible constructible figures in single-fold paper folding. The original 6 axioms, more appropriately termed *folding operations*, introduced in 1989 are credited to Humiata Huzita. A seventh was introduced in 1991 by Jacques Justin. Together they are termed the Huzita-Justin Axioms (HJA). For reference, Figure 4.9 shows 6 of the 7 axioms. It is shown by Roger C. Alperin and Robert J Lang the original 6 axioms fully describe all possible single-fold operations, and that the 7th axiom is a special case which did not resolve any more solutions. Alperin and Lang further proved that there are no further axioms and resolved its completeness. In fact, working independently, Martin showed that the operation equivalent to Huzita's 06 axiom (plus the definition of a point as a crease intersection) was, by itself, sufficient for the construction of all figures constructible by the full six axioms and that this included all compass-and-straightedge constructions (in Euclidean geometry).[88] This is significant primarily for the reason that all folds used for the case studies in this thesis are based with the single fold operation in mind.

[87] Robert Lang, "Folds Way New Origami," (Filmed February 2008, TED, Posted July 2008).
[88] Roger C. Alperin and Robert J. Lang, *Origami 4: One-, Two-, and Multi-fold Origami Axioms* (Massachusetts: AK Peters, Natick, 2009).

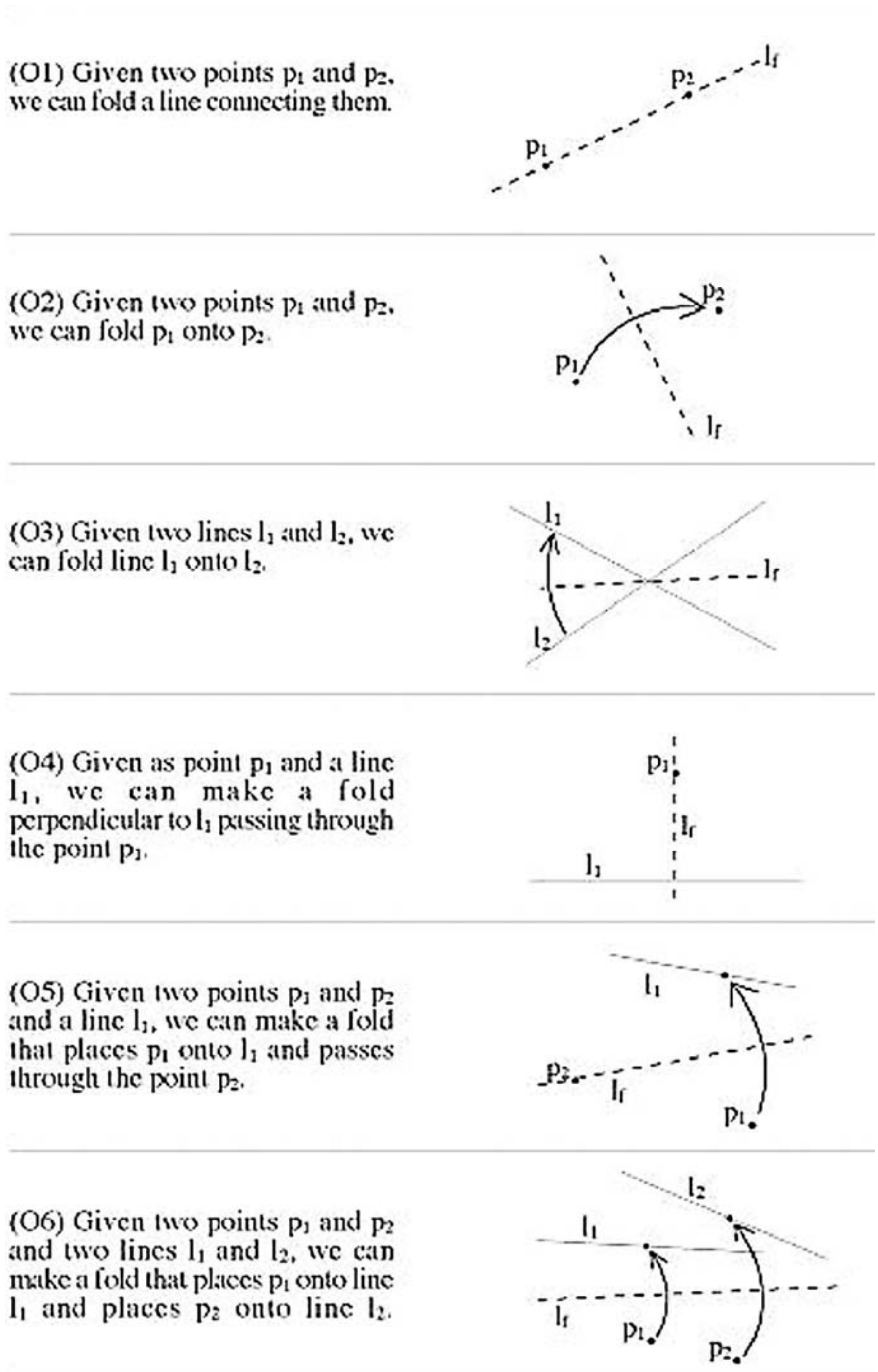

Figure 4.9. 6 Folding Axioms[89]

Paper folding can be reduced to these 6 proven folding principles.

George Martin further simplified them into one folding rule based on the 6th axiom.

[89] Robert J. Lang, "Huzita-Justin Axioms," Last modified November 11, 2009, http://www.langorigami.com/science/math/hja/hja.php.

SPACE GRAPH

MATRIX

ch5

5. MATRIX: GRAPHS

Man's mind, once stretched by a new idea, never regains its original dimensions.

Oliver Wendell Holmes (1801-1894)

Paper folding can be translated into a collapsed form using a concyclic quadrilateral graph or diagram. This thesis interchangeably uses the term *shells.* As an existing method in projective geometry, concyclic diagrams use simple polygons to understand geometric transformations where a circle is used as a circumscribing element of any polygon. The information from the transformations of the polygon is plotted on points of the circle. The points represent the information of the transformations between 2-D and 3-D. Each fold has its own diagram which forms the slices of the folding process. A central axis is the reference point for the angles and time, as paper folding is a time-dependent process. The folded forms can be reconstructed from that information into a matrix.

The graphs are taken through the entire paper folding cycle, from undifferentiated sheet, to completed fold, to its unfolded state with resultant crease. The information is strung together in two ways: 1) the points along the circle graphs; and 2) through the central axis, which represents time. The 2-D format of the graphs allows the paper folding to be represented as instances in time of the folding process, not slices of the folding itself. In a flat fold the geometry of the information is not completely flat, because each fold is paired with another layer, preserving the constraint that the paper cannot penetrate itself. The author of this thesis also developed the idea of giving each side of the paper two values or phases: + and -, essentially denoting the front and back of the sheet of paper. Figure 5.1 shows how the square sheet is plotted on the concyclic graph or shell graph. The graphs allow paper folding to be plotted in top 2-D view as well as a frontal profile view. The shell graphs have four quadrants which relate and to which all of the vertices are plotted on. As each new fold is made, a new numerical designation is given to the new set of points. If any of the plots of the vertices remain the same the then that information carries over into the next graph.

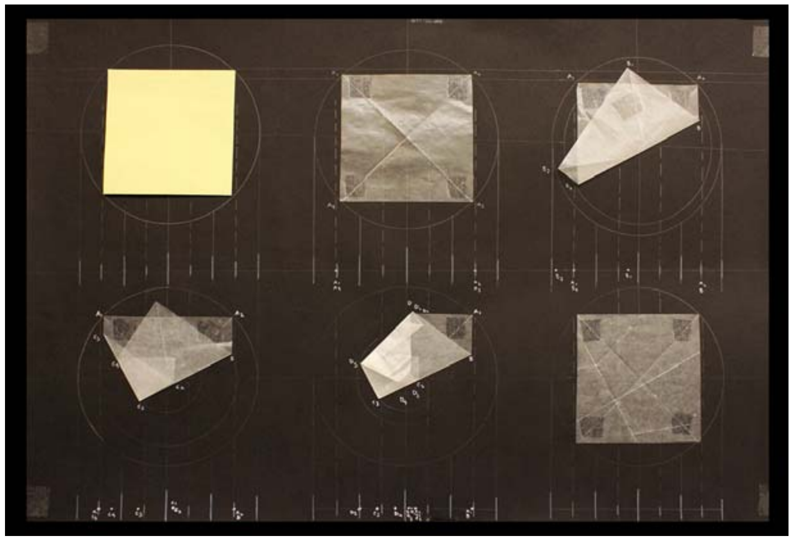

Figure 5.1. Draft of Concyclic Diagrams[90]
The diagram show six steps of an arbitrary fold using a concyclic diagram to plot the points of geometry of the folding process. The data can be used to recreate the spatial configuration of the folded object.

A basic concyclic or shell graph is shown below in Figure 5.2. Concyclic diagrams are an existing method in geometry to study polygons. The method is adapted by the author for use with paper folding and is not found in existing literature dealing with paper folding. Each graph of the three different types of paper folds begins with an undifferentiated (unfolded) sheet of paper as in *Graph 1*. The sheet has five colored lines which represent the hyperplanes or mirrors. The sheet of wax paper is approximately a 6 inch by 6 inch square. All sheets used in this experiment are cut from an identical cutting template. Each line on the paper has a half arrow which denotes the top-down orientation, facing side, and handedness of the sheet. There are five colored lines, each spaced 1 inch horizontally from each other. The black line represents the primary mirror. The blue, green, red,

[90] Graphs by author.

and purple lines represent secondary mirrors, and are colored to track the different parts of paper during the folding process. The sheet of paper is oriented according to the central axis on the graph. As the folds proceed through each stage the center of the paper is always positioned at this point. Each graph has four quadrants according to geometric convention, and run counter-clockwise: A, B, C, and D. The graphs are divided into two hemispheres: top and bottom. There are two different types of graphs. One is shown by the overhead view with the concentric circles. The graph below each circle graph is a profile of that graph. The larger primary lines represent the bottom or blue hemisphere, and the smaller set of secondary lines the top or pink hemisphere. From Figure 5.2 the unfolded state of the paper is represented by the number "1". For example, in Graph 1 all points are at the same layer 1: A1, B1, C1, and D1. The points are also related by which hemisphere they are in. Both the alphanumeric and colored points follow a stacked system. When each new fold is made a series of new data points are made to the existing set. However, as can be seen through the Graph 1 through 4 the original points are not necessarily retained as the paper folds into different sections in the shell graphs. The graphs essentially record the information through the vertices of the folded paper along the circle. As the fold progresses the sheet usually gets smaller in scale. However, this feature is taken into account because the center axis of the paper remains consistent on the graph.

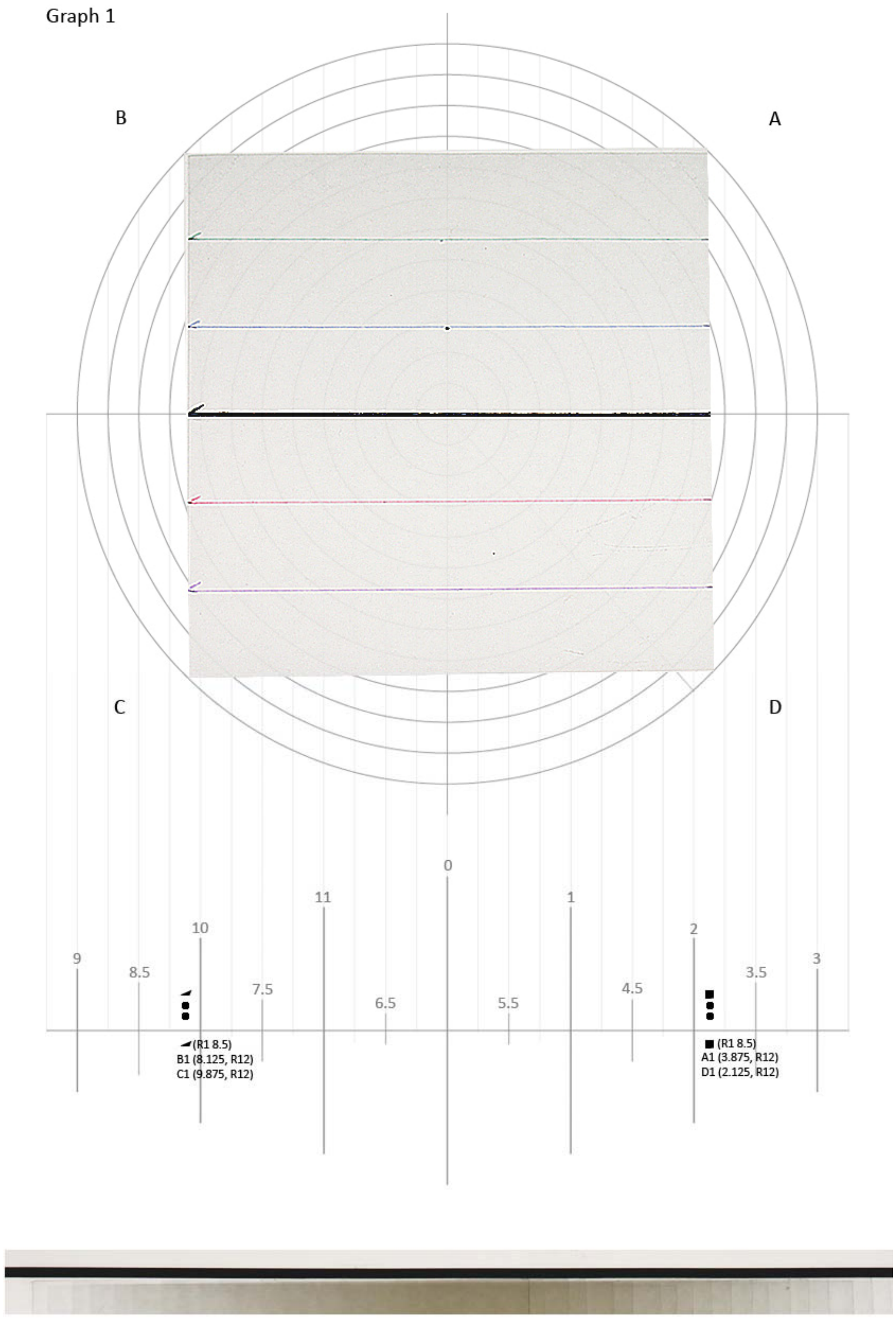

Figure 5.2. Concyclic Diagram[91]

[91] Graph by author.

SPACE GRAPH

MEASUREMENTS ch6

6. MEASUREMENTS

> If a thing can be observed in any way at all, it lends itself to some type of measurement method; No matter how fuzzy the measurement is, it is still a measurement if it tells you more than you knew before; And those very things most likely to be seen as immeasurable are, virtually always, solved by relatively simple measurement methods.[92]

Measurements of space are made by comparing the ratio of the distribution to a value along the shell graphs. The observations are made through a series of random samples of two different types folds. In the first example five samples of an irregular fold are plotted. In the second example 10 samples of a regular pattern fold are studied. The measurements may include sources of error in several forms. As a means to study the subtle characteristics of paper folding this thesis uses hand folding for the sheets of paper used in the shell graphs. Mechanical or digital means of folding either do not have the dexterity nor subtle characteristics of physically folding the medium of paper. However, physical folding can create systematic errors which then follow through the sequence of folds. This may affect the information in the shell or concyclic graphs. The graphs themselves may also not be fully representative of the folding process, and although there are many unknowns in the complexities of folding, an attempt is made to retain as much of the relevant information as much as possible.

The graphs show an important characteristic about paper folding and space: the perception of space is both continuous and discrete; as such its configuration with physical form can be measured. Discrete space and continuous space are alike in this respect: they divide into intervals

[92] Douglas W. Hubbard, *How to Measure Anything* (New Jersey: John Wiley & Sons, 2010).

which are their parts.[93] In the context of this thesis the circle used for the graphs and the square sheet of paper itself may seem finite, but they can be considered infinite entities. This feature allows both the graph and paper fold object to be consistent with space having a continuous property. The circle can be considered a projective space, in our case a 2-D one, that has an infinite amount of points.[94] In the graphs, a series of concentric circles are used to plot the vertex points of each consecutive fold. Each point contains information relating to both the position of the physical object and the distribution of space particular to any particular configuration. Paper folding also exhibits a similar property of continuity and infinity as described earlier in Chapter 2.

The first example folding case study is below. Gray dot represent inherited fold from a previous graph. Black dots represent points from a new fold. The arrow denotes the front of the primary hyperplane line and the side of the paper. And the square shows the tail-end of the primary line. The first set of data points at the lower left corner of Graph 1 read as such: (R1 8.5), B1 (8.125, R12), and C1 (9.875, R12). The first point is the front of the primary hyperplane and is denoted by the letters R or AR. The number 1 signifies that it is part of the first fold. A second fold for example would be R2 if the point moves to a different location. The number 8.5 shows where the point is along the series of concentric circles 'R', 0 being its center, and 12 being the outermost circle. The second point set denotes that the point is in quadrant B and is from the first fold 1. The number 8.125 shows where the point is plotted along the profile graph, which is below the circle graph. R12 shows that the point is at the outermost circle. The third set of numbers show the point in quadrant C of fold 1. Along the profile graph it is found at 9.875 units and at the 12th concentric circle. Only a few samples are taken from each of the two case studies due to time requirements. Case Study 1 immediately follows.

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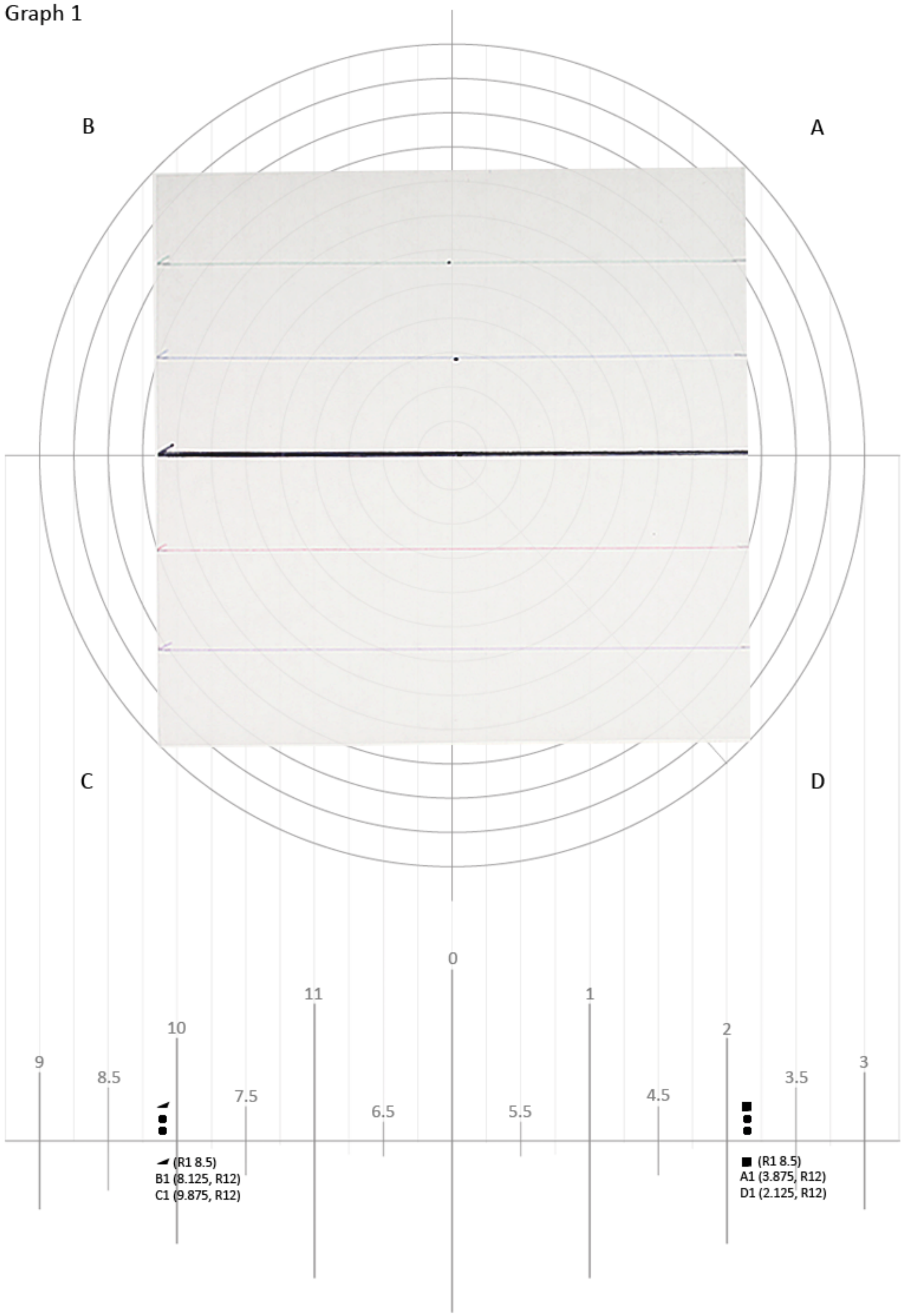
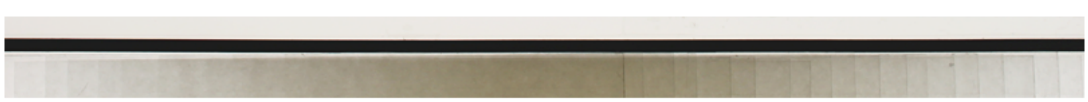
[93] Graham Nerlich, *The Shape of Space* (Cambridge: Cambridge University Press, 1994), 203.
[94] Ibid, 90-91.

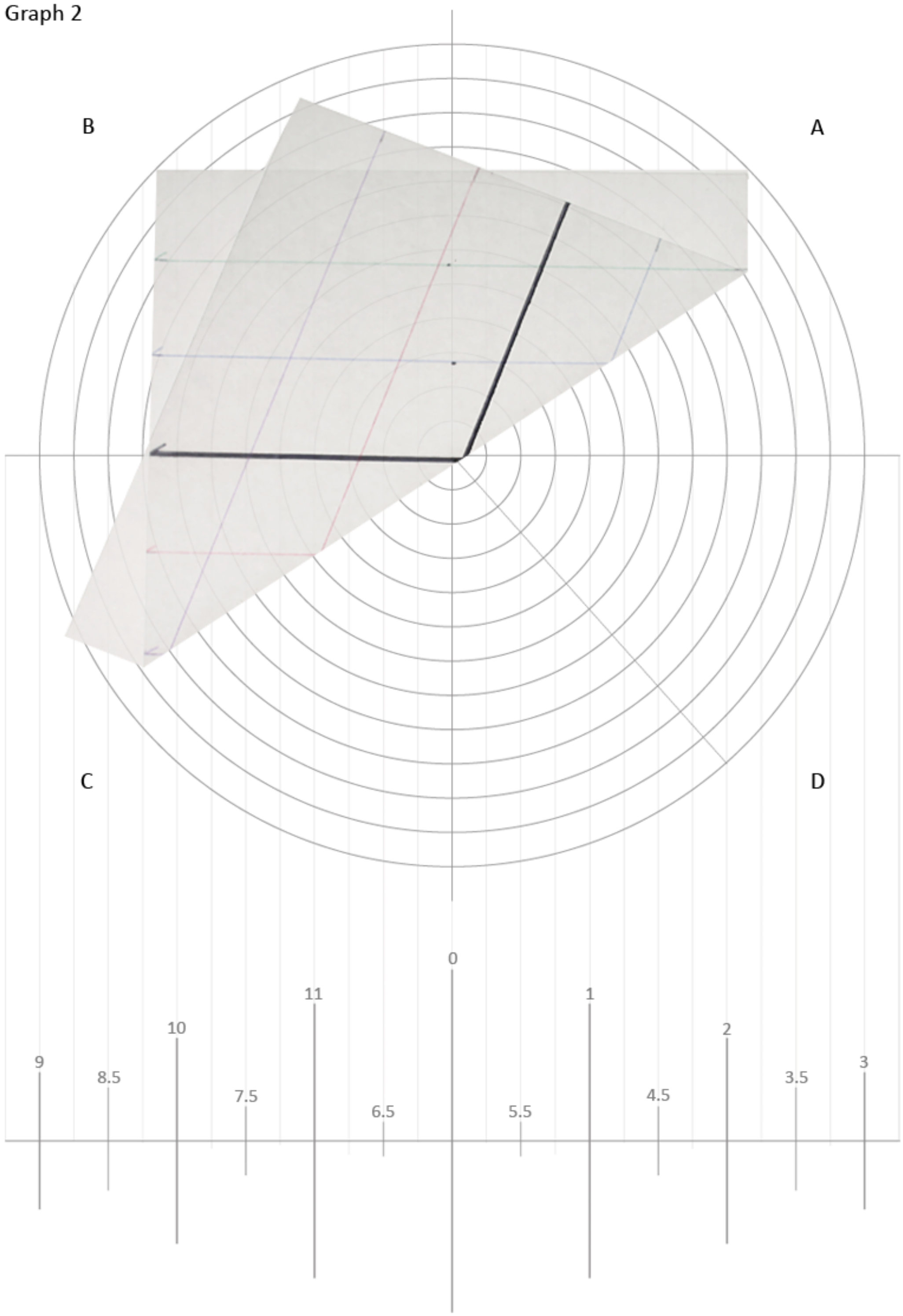
Graph 1

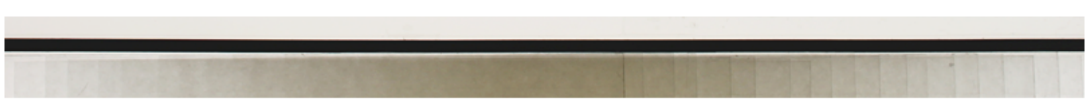




NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

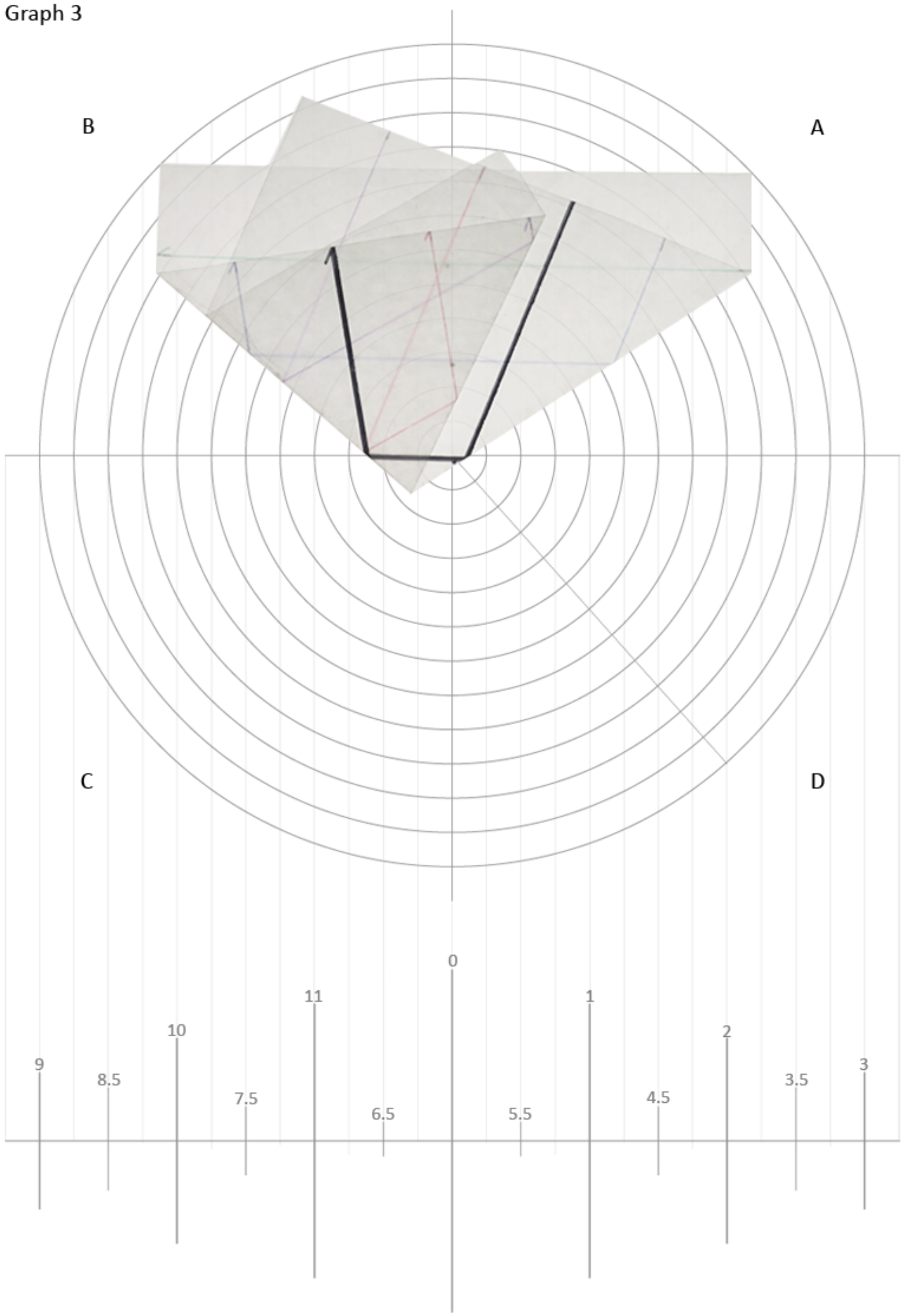
Graph 2

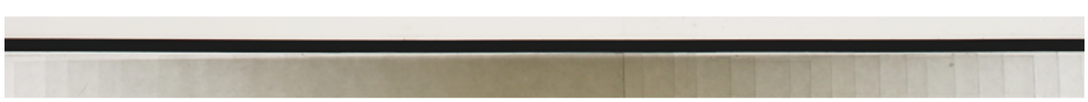




NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 3





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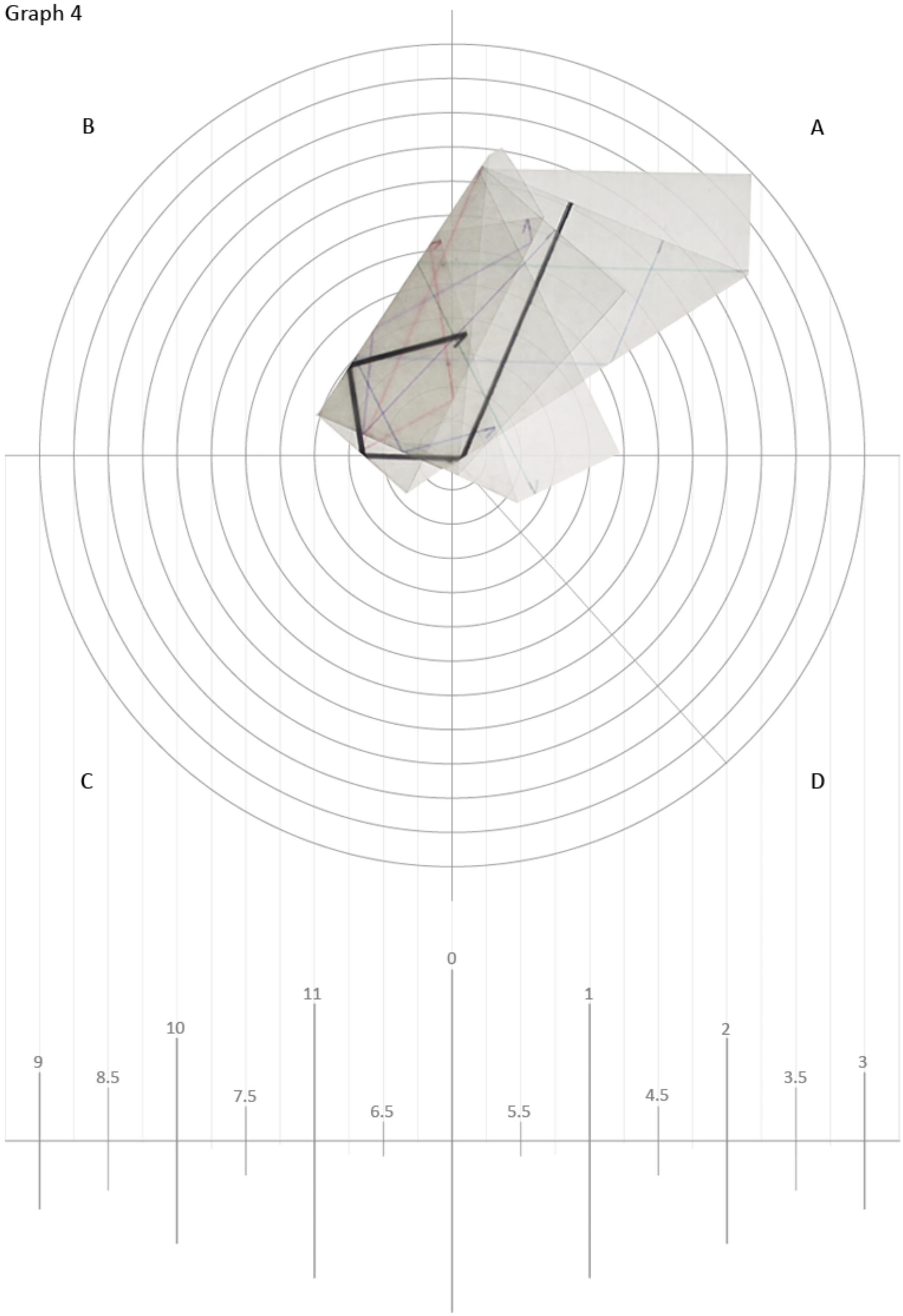
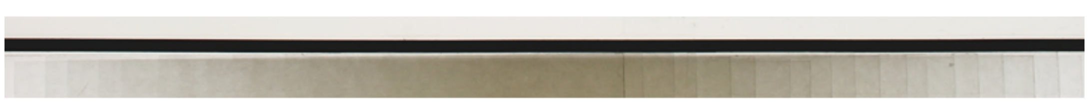
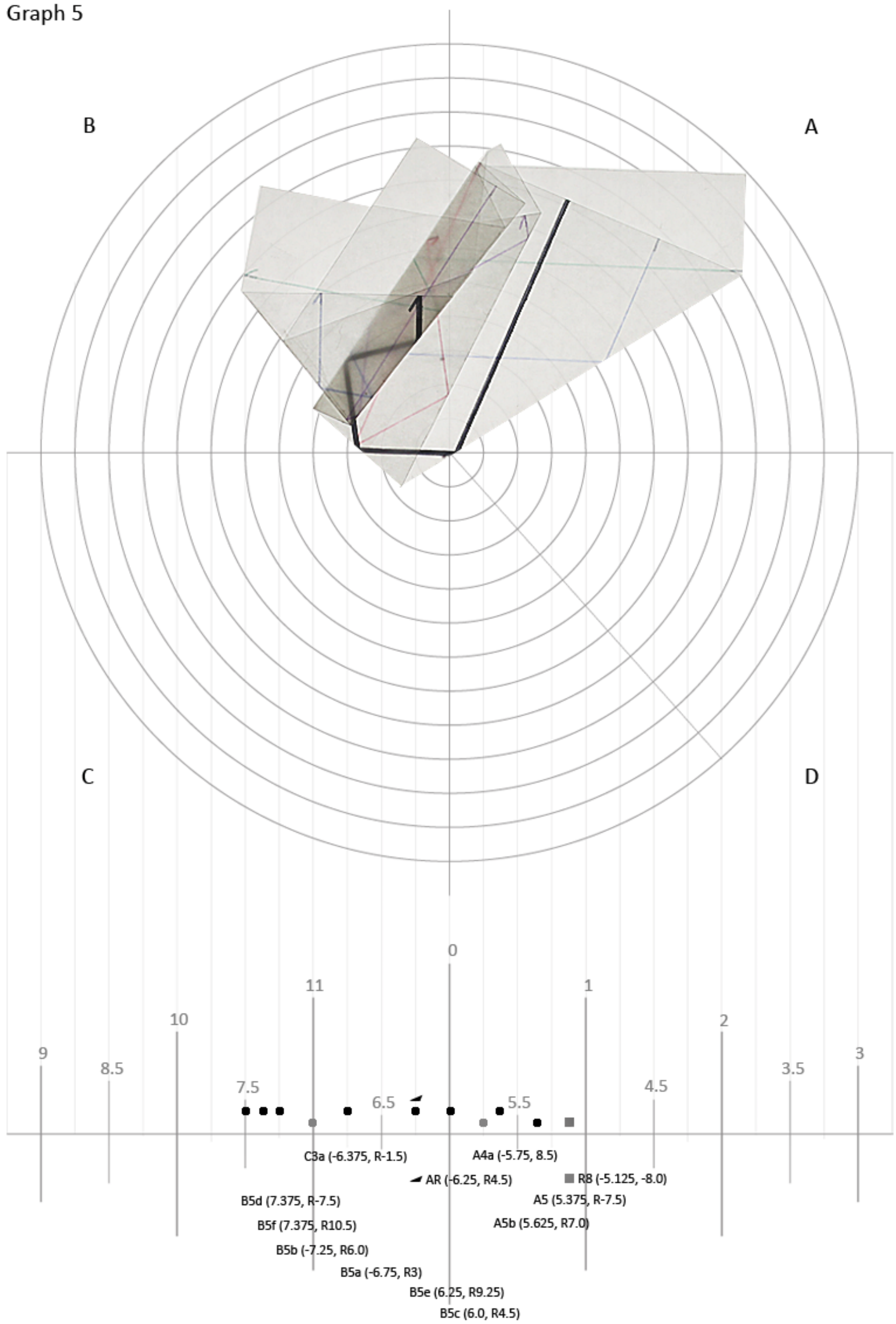
Graph 4





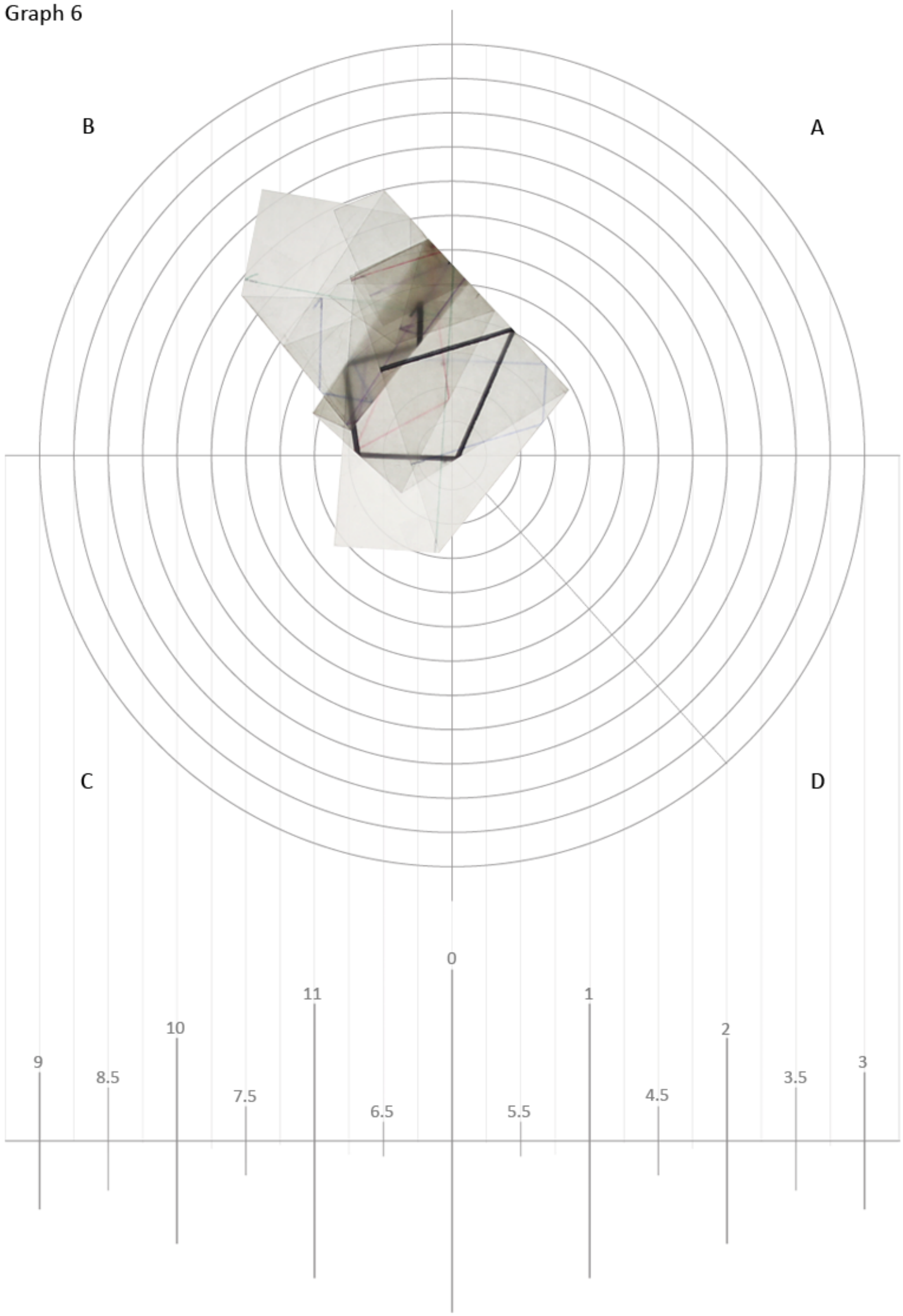
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

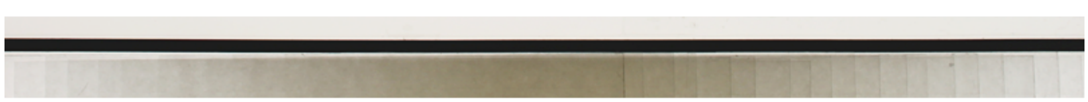
Graph 5



NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 6





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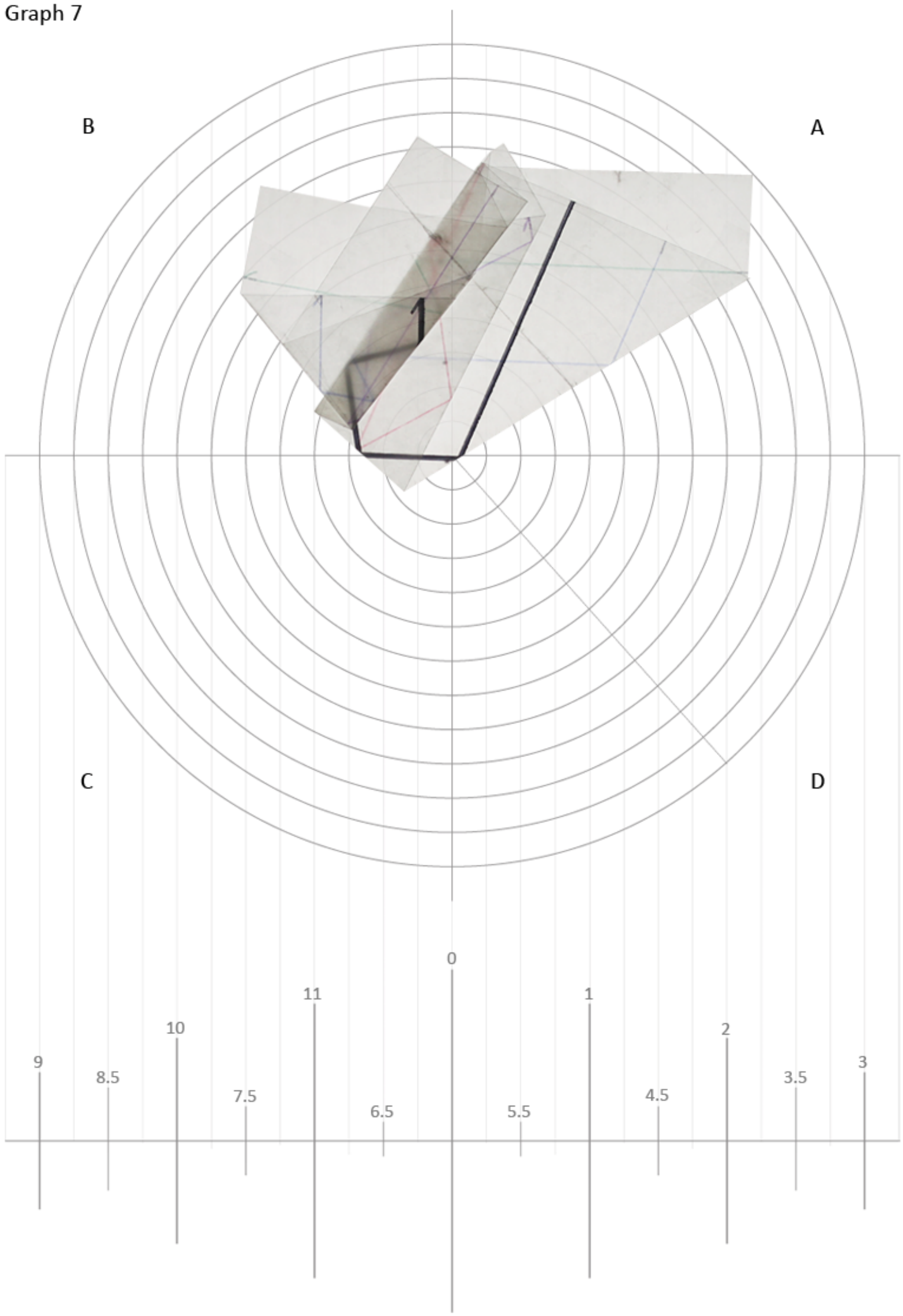
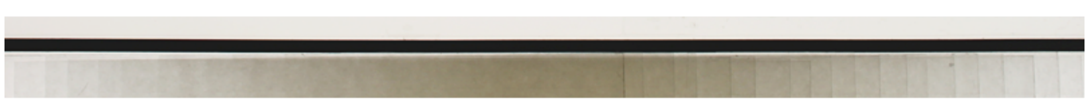
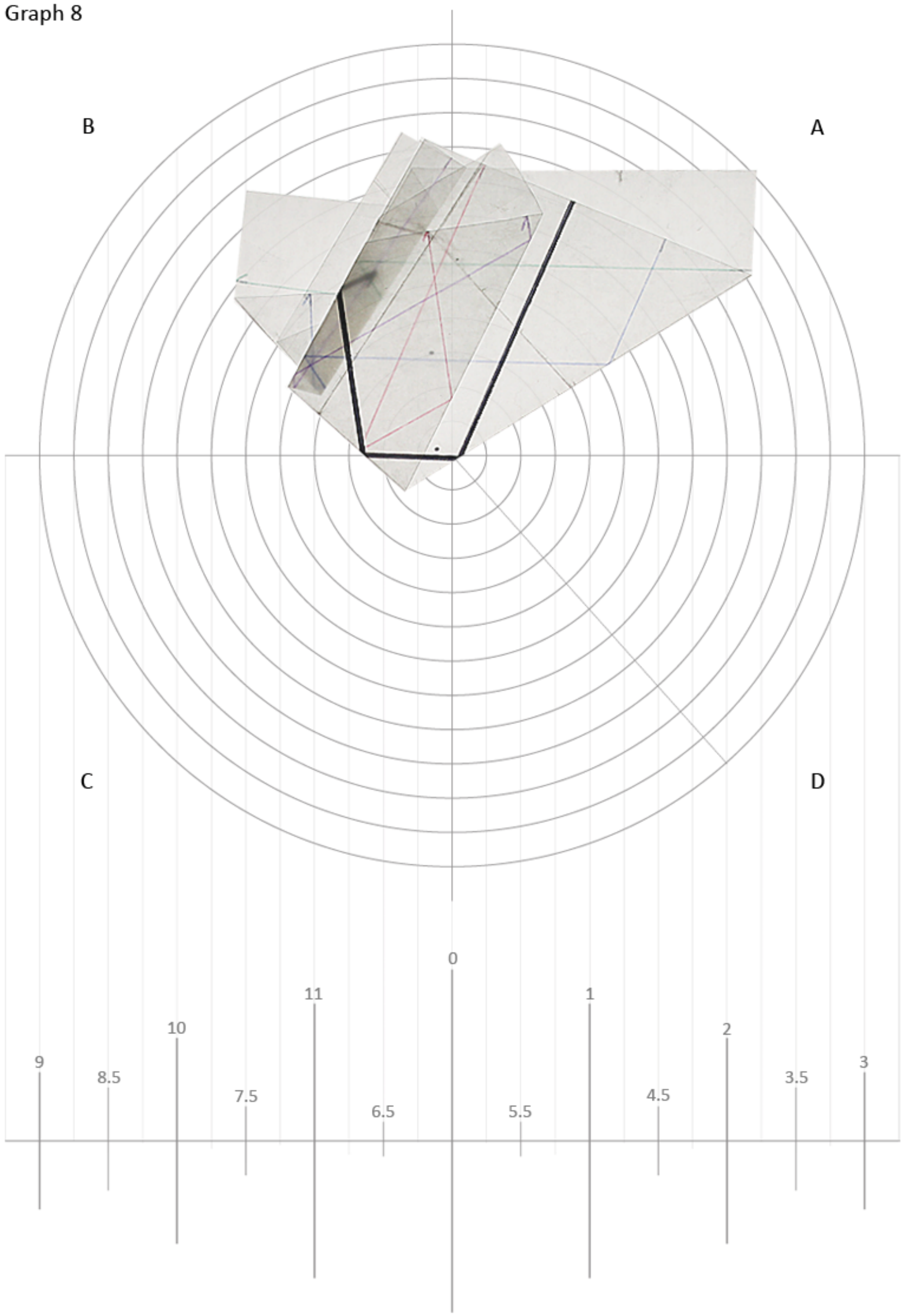
Graph 7

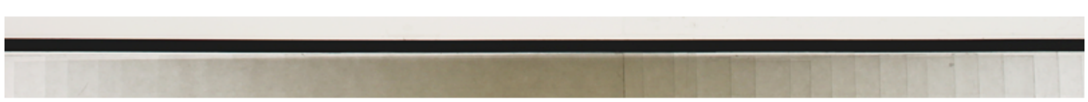




NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

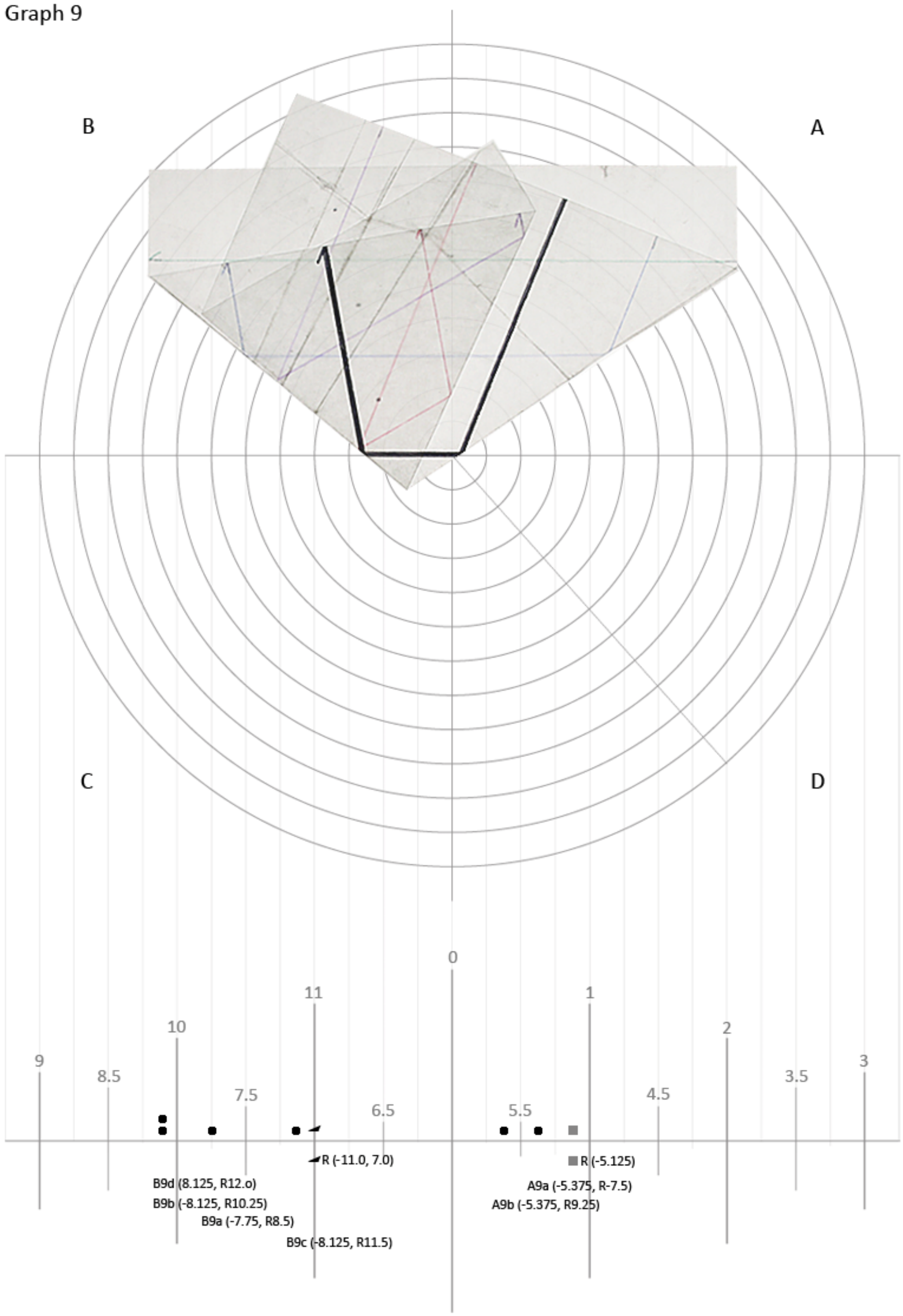
Graph 8





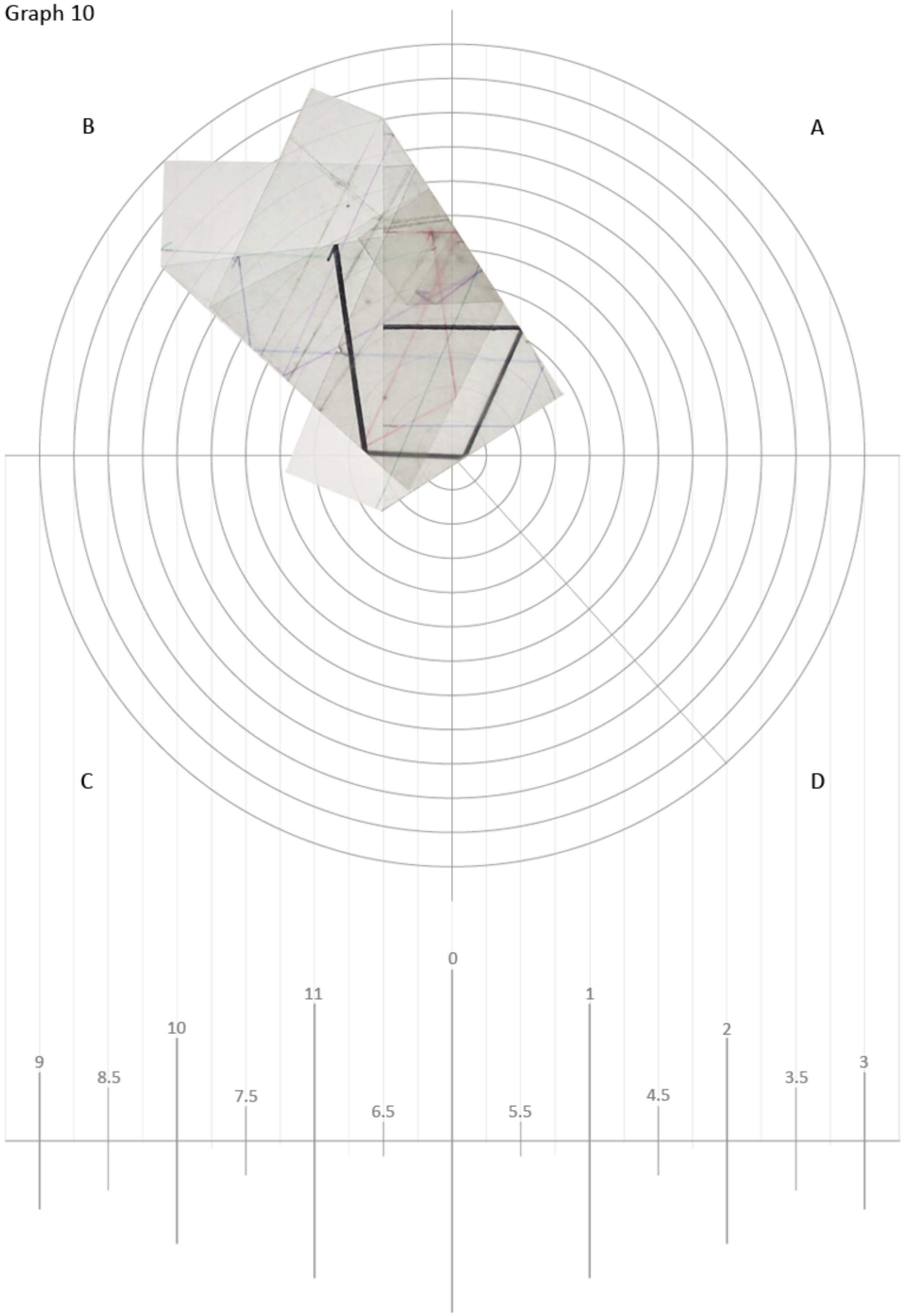
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

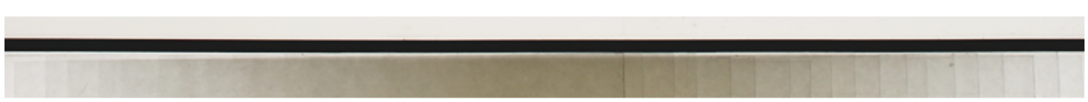
Graph 9



NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

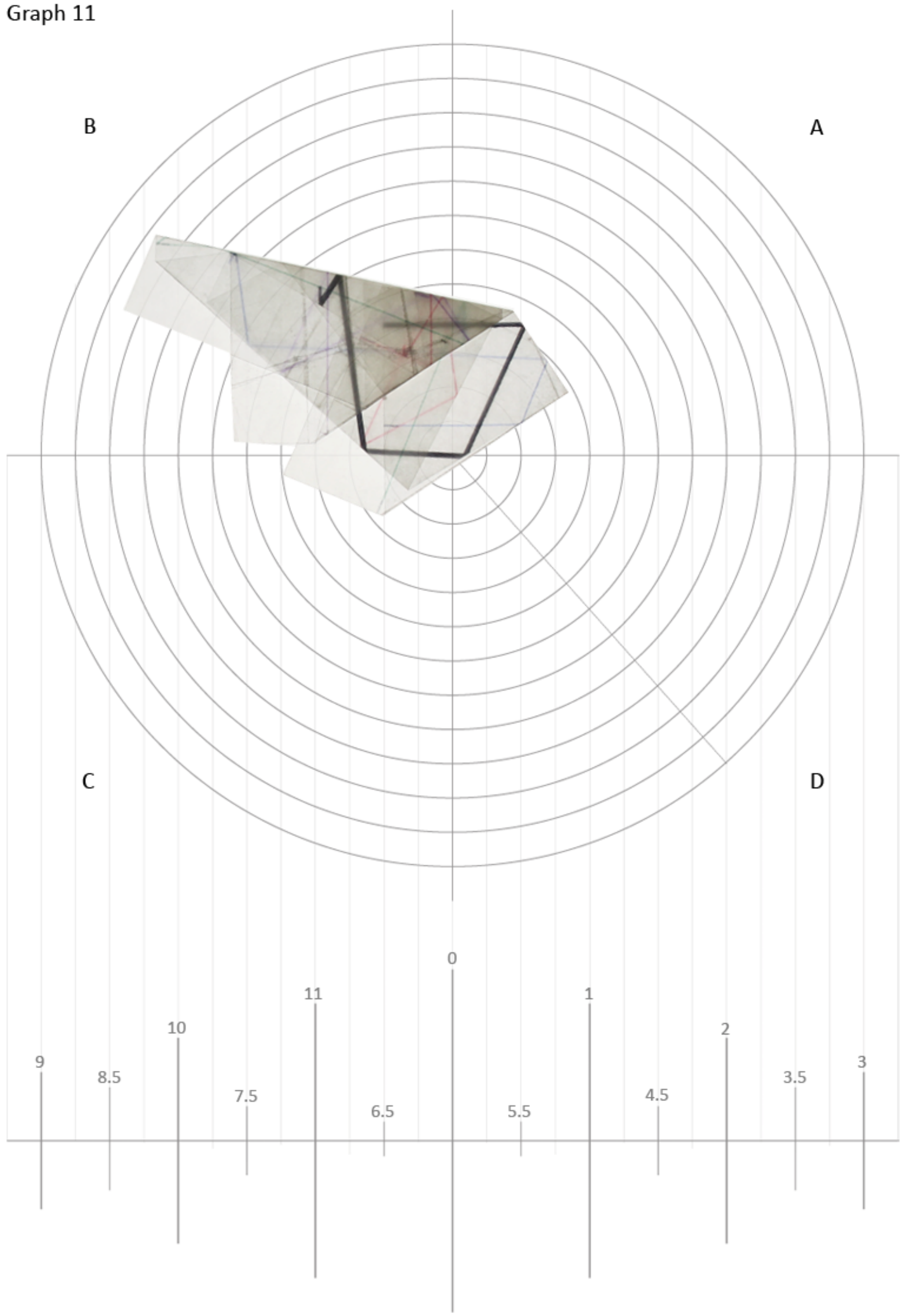
Graph 10





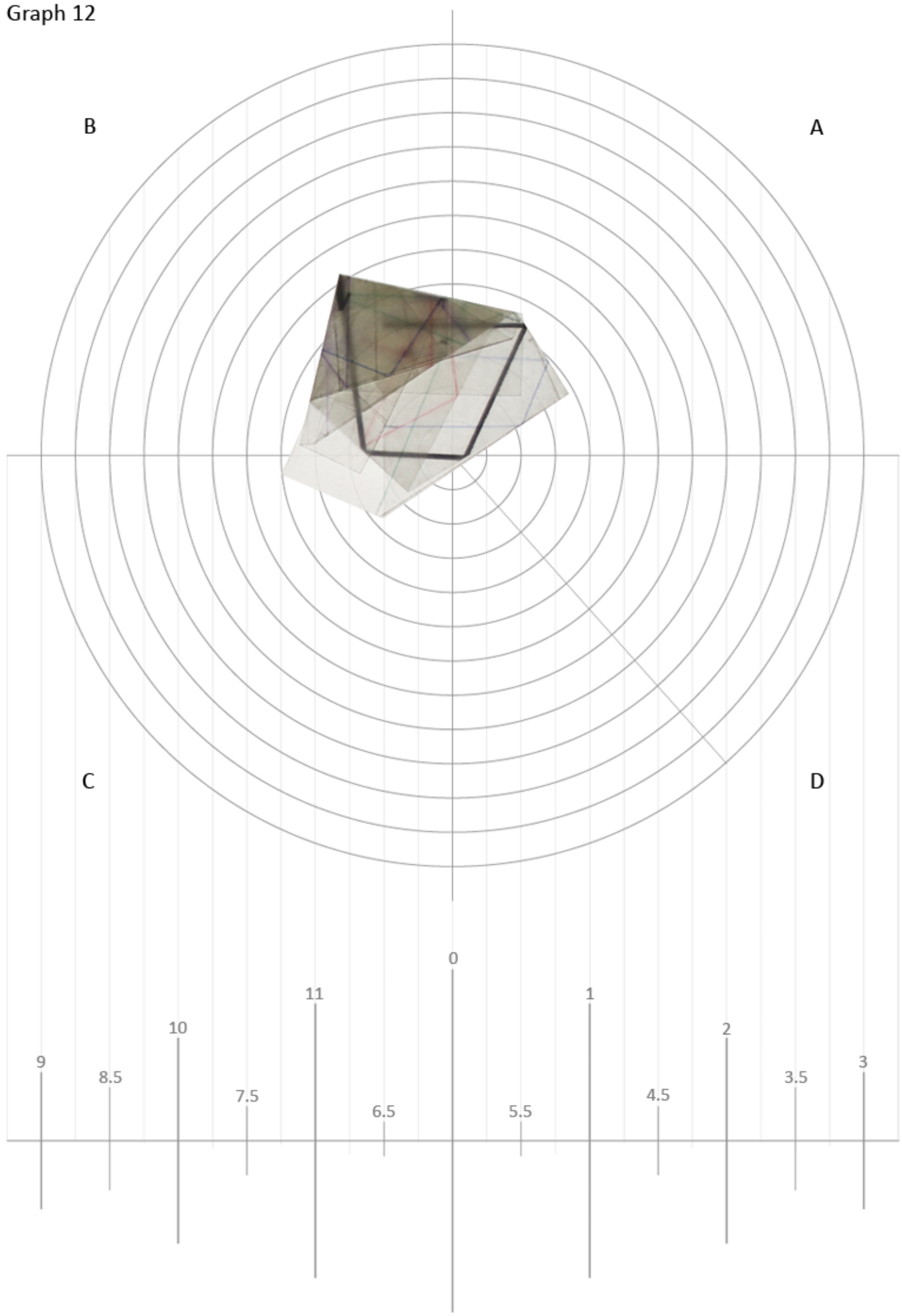
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

Graph 11



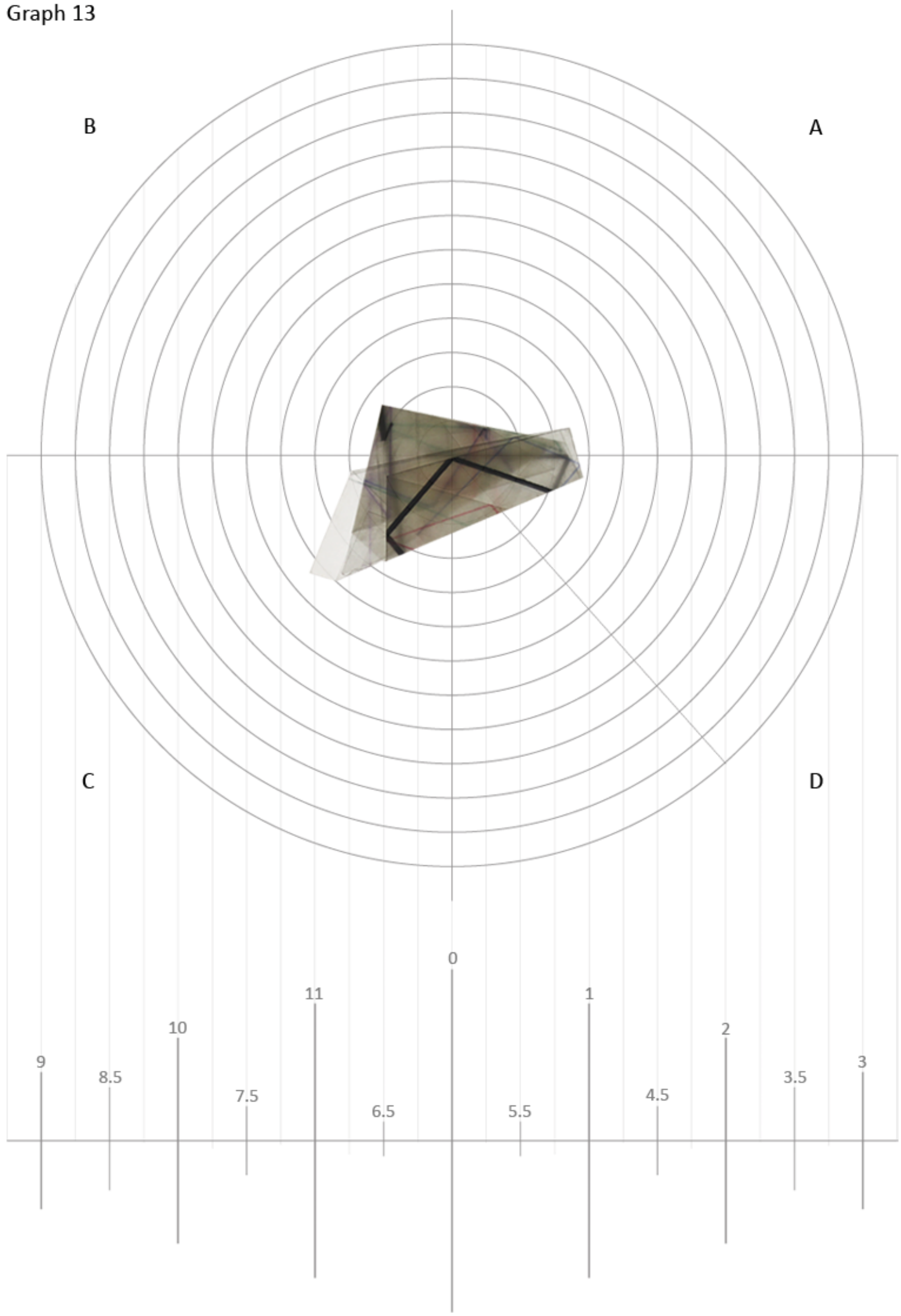
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 12



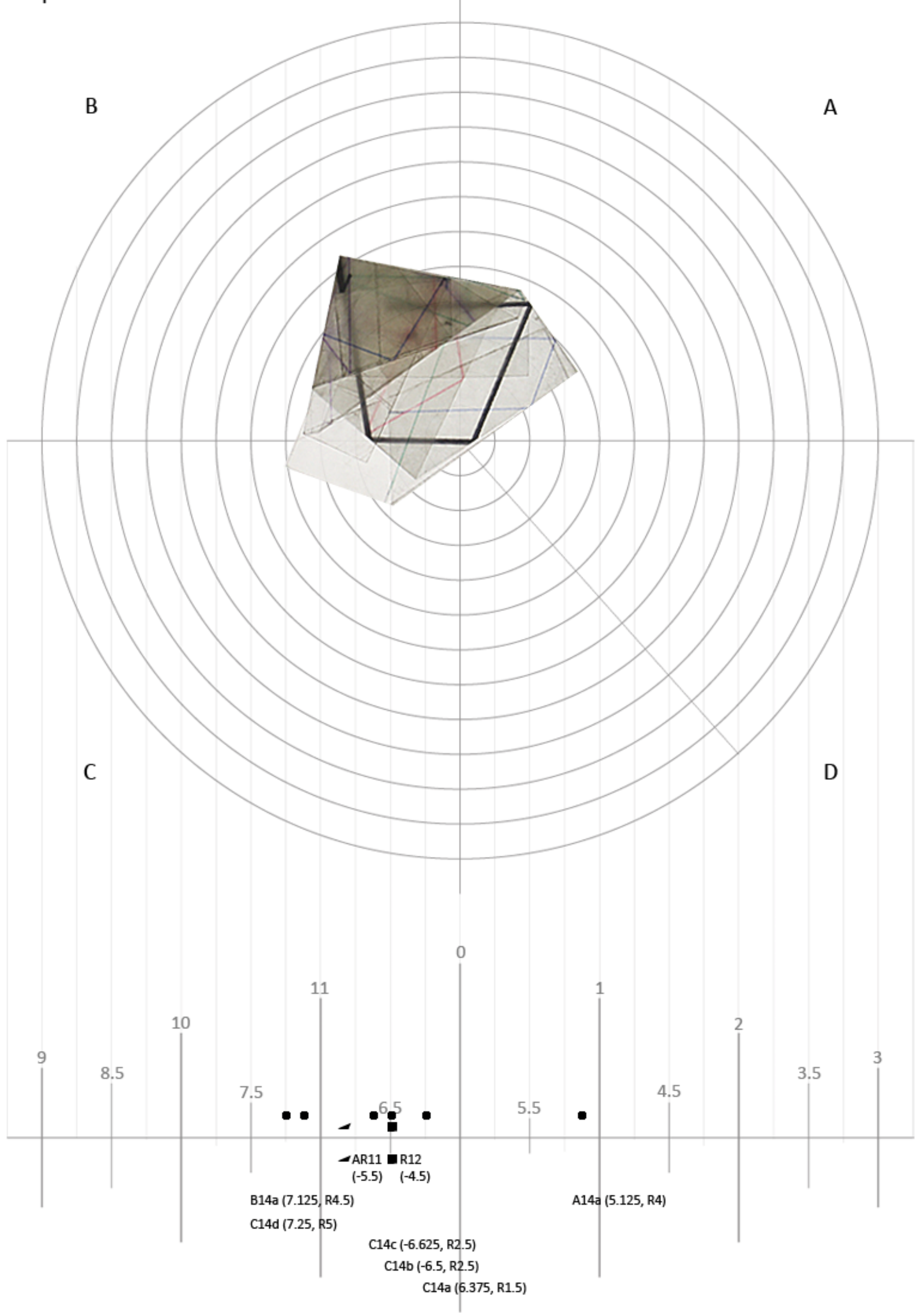
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

Graph 13



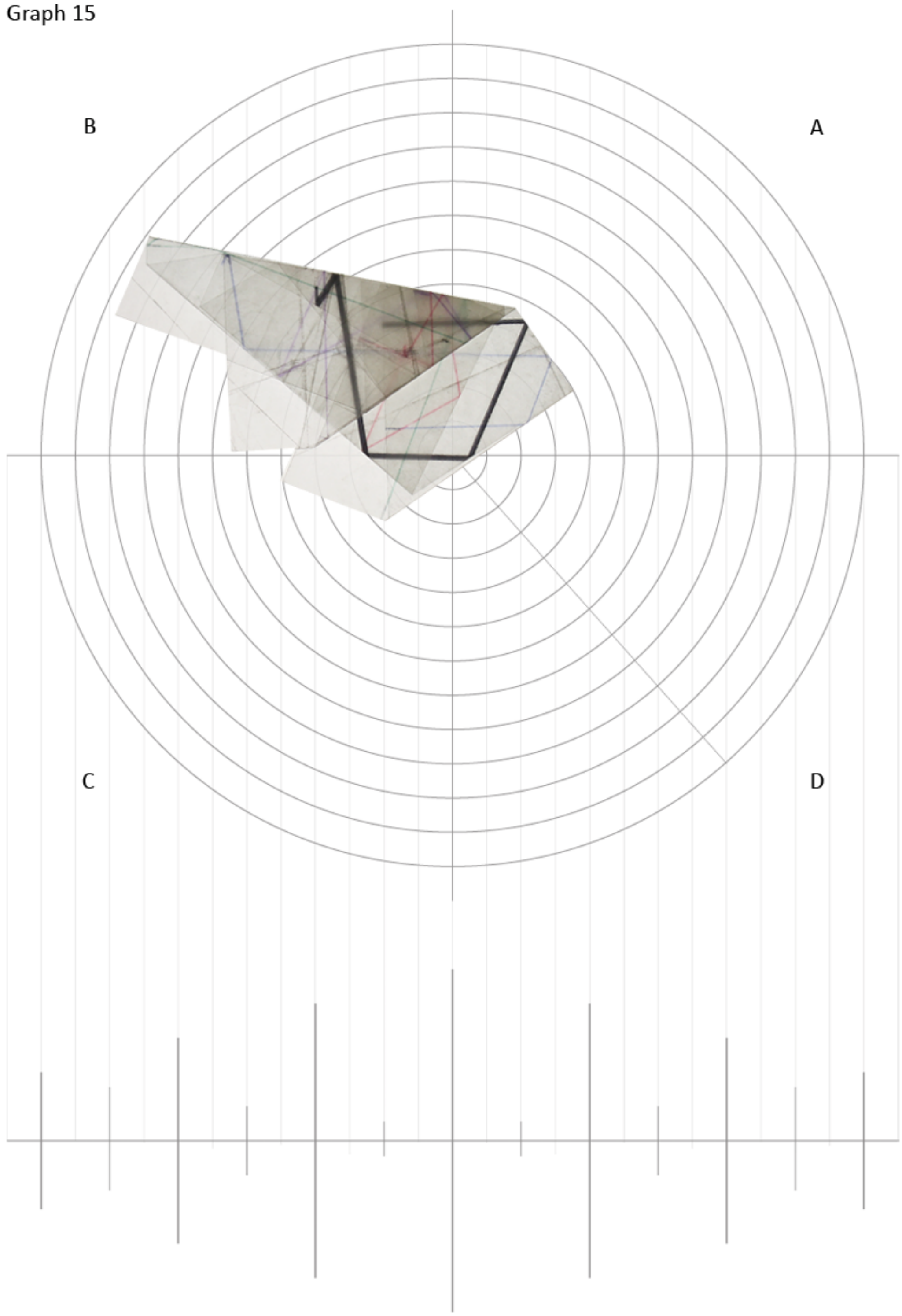
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

Graph 14



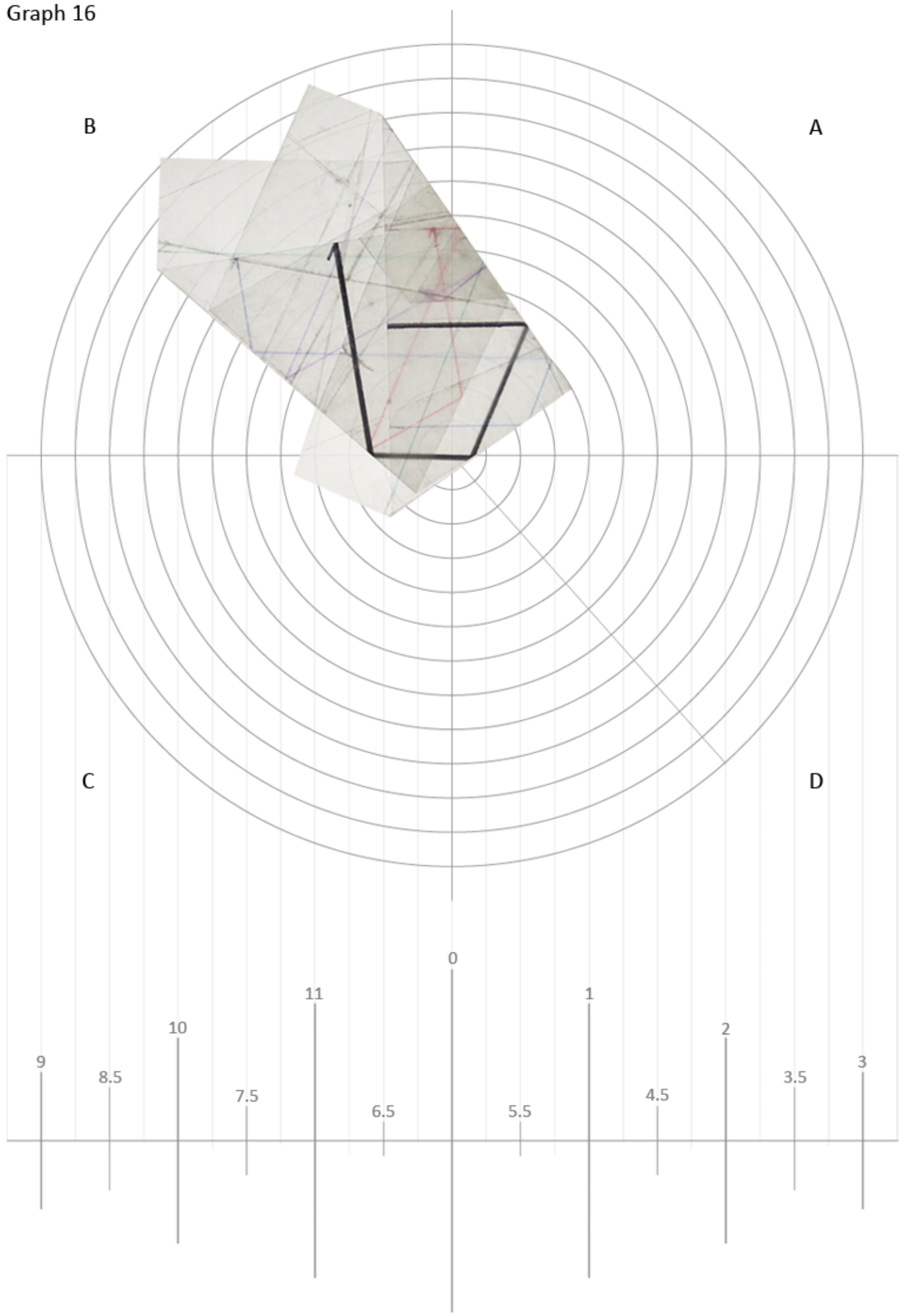
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

Graph 15



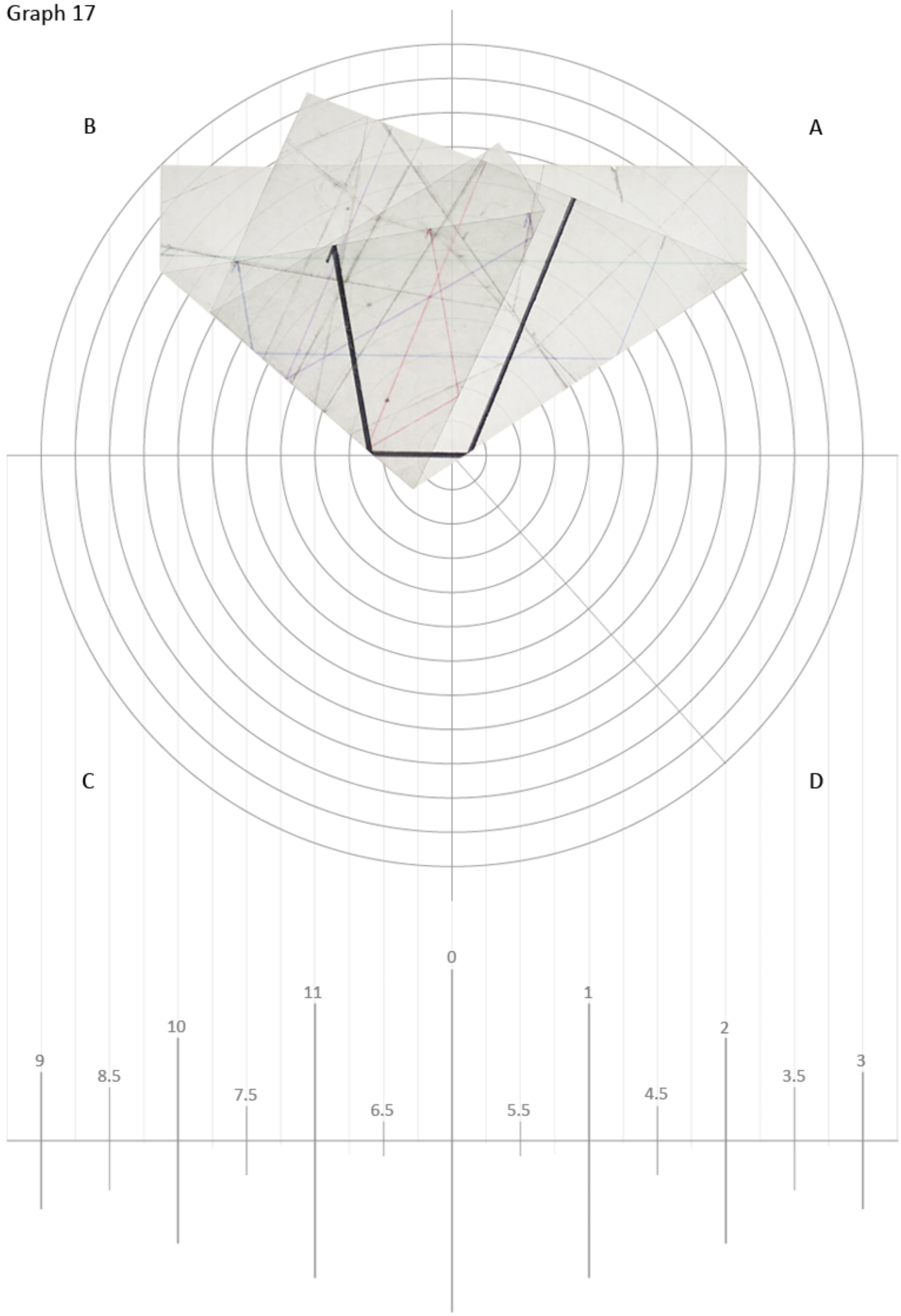
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

Graph 16



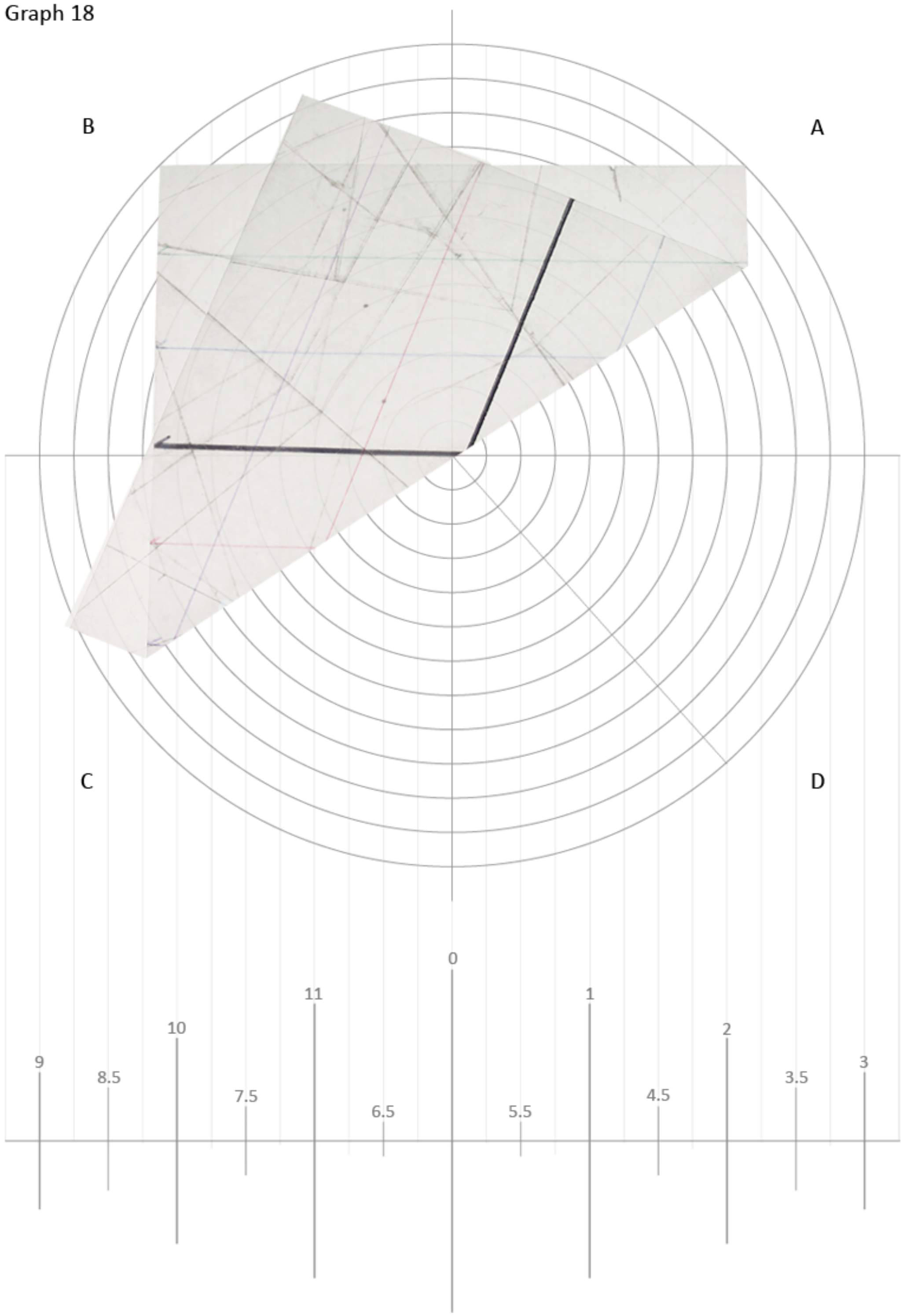
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

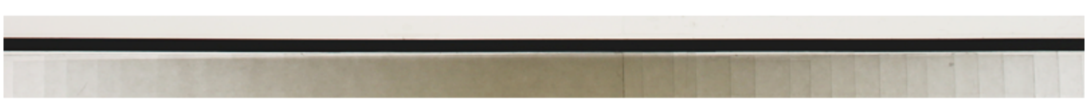
Graph 17



NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

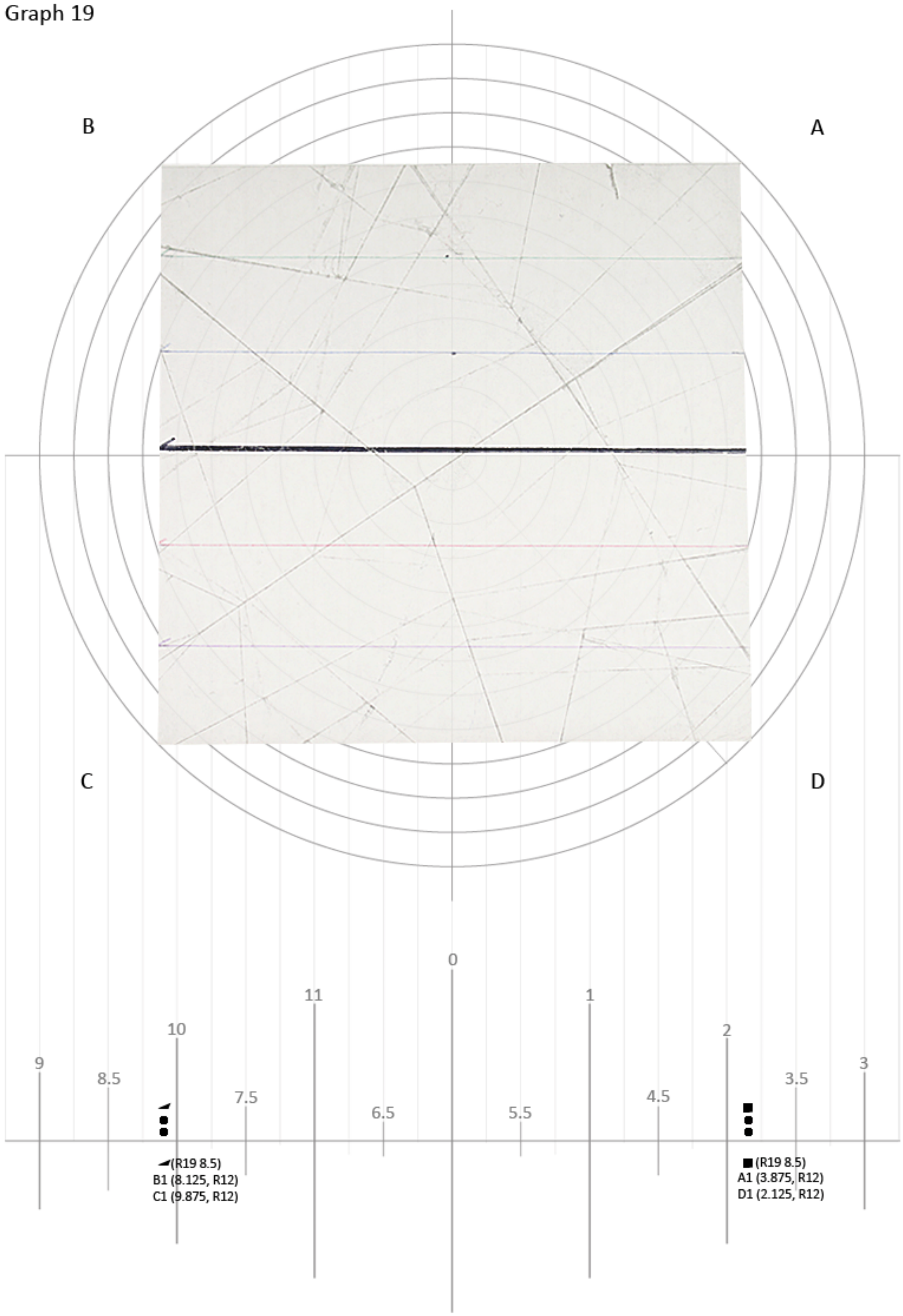
Graph 18

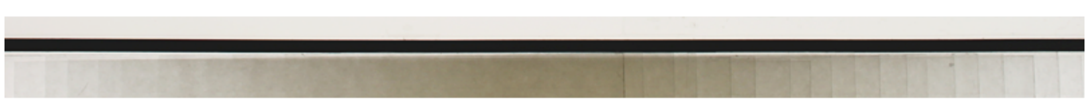




NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

Graph 19





NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

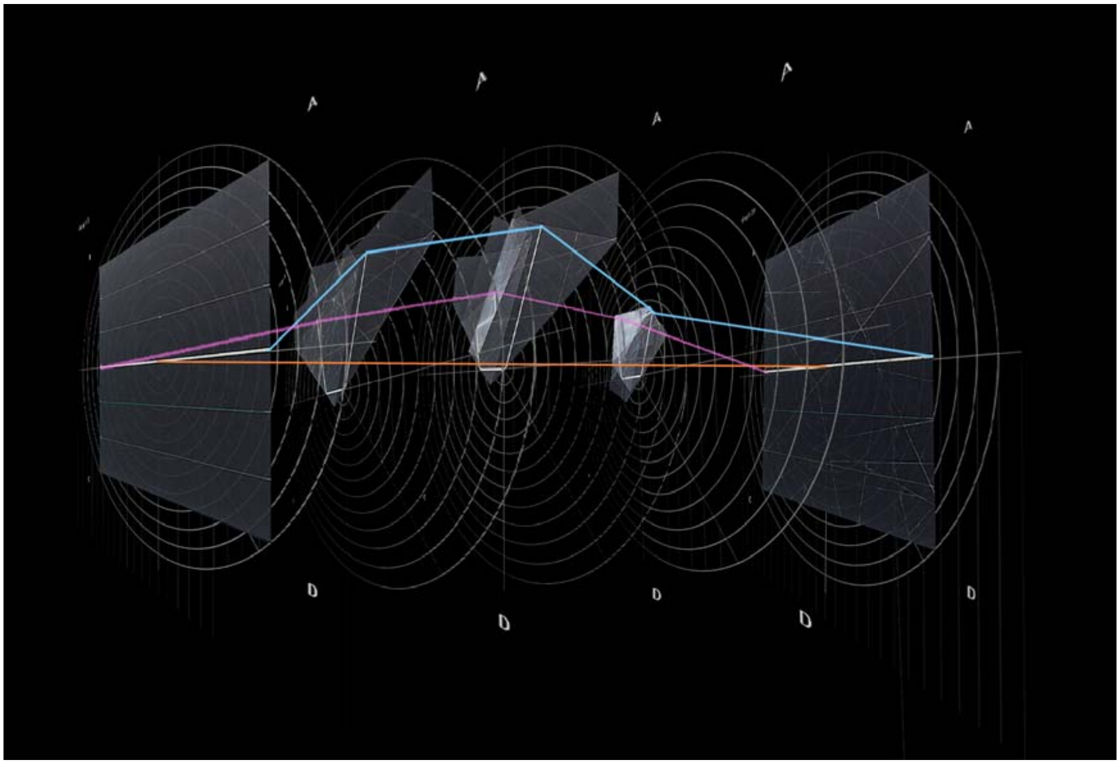
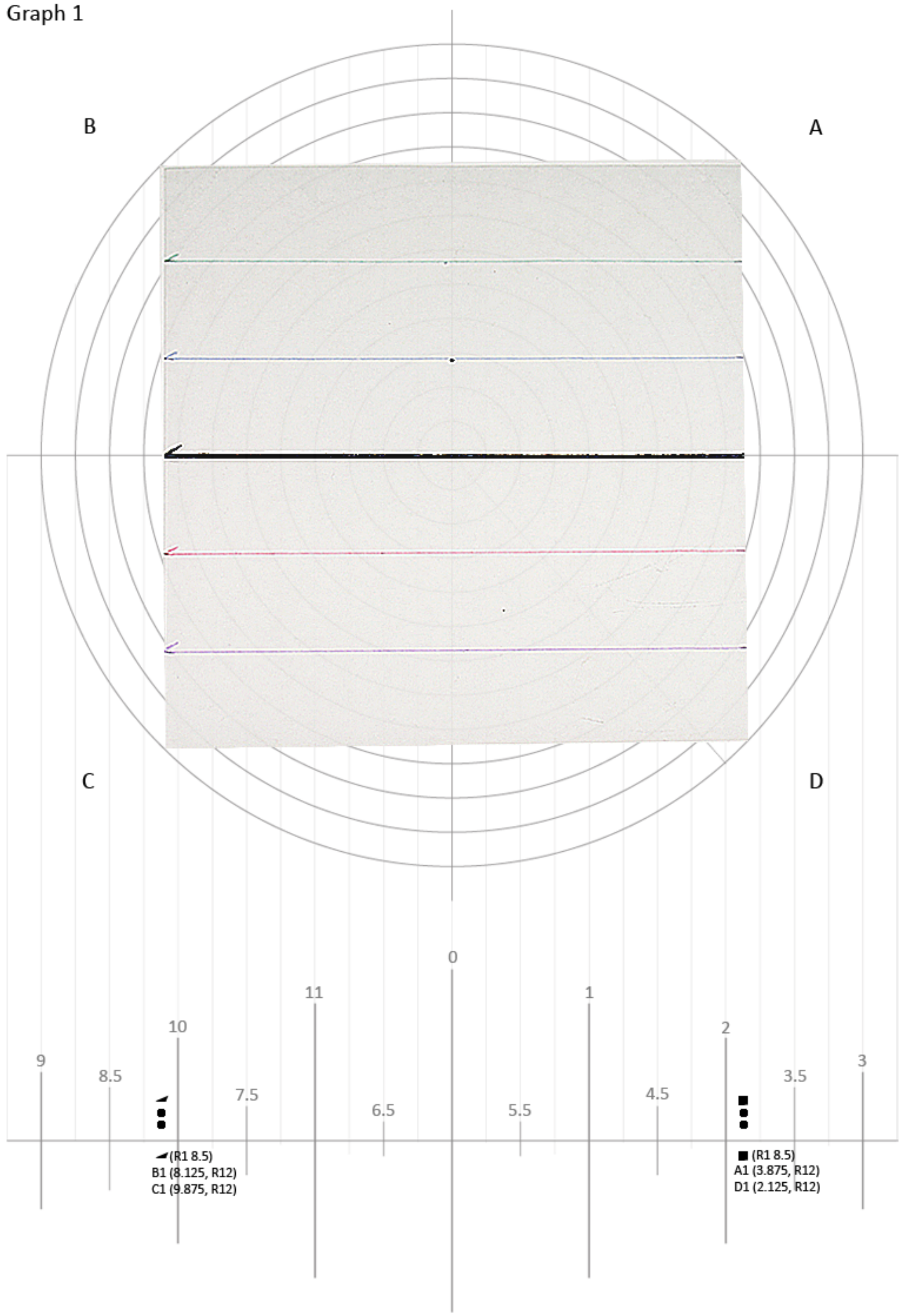


Figure 6.1. Hyperplane Projection: Paper Folding Graph 1 Timeline[95]
The diagram maps out the series of the samples taken from Case Study 1 in 3-Dimensions to study the behavior and structure of space during the folding process. The blue line represents the right end of the primary hyperplane, the pink shows the left end, and the orange line describes the central time axis.

The most obvious visual quality is how the data points compress through the paper folding cycle. What is less obvious is that regardless of the type of fold, its position on the graph, or how compressed a folding pattern is, it remains a square sheet of paper. The second example folding case study is below. In certain graphs folds may not carry over into the next generation graph, especially if the sheet is unfolding. Figure 6.1 shows the connection between the hyperplane points of Graphs 1, 5, 9, 14, and 19. The blue line represents the points closest (the endpoints of the primary hyperplane line) to the viewer and the pink line (the front end of the primary hyperplane line) show the points farthest away in general. The orange line is the central time axis.

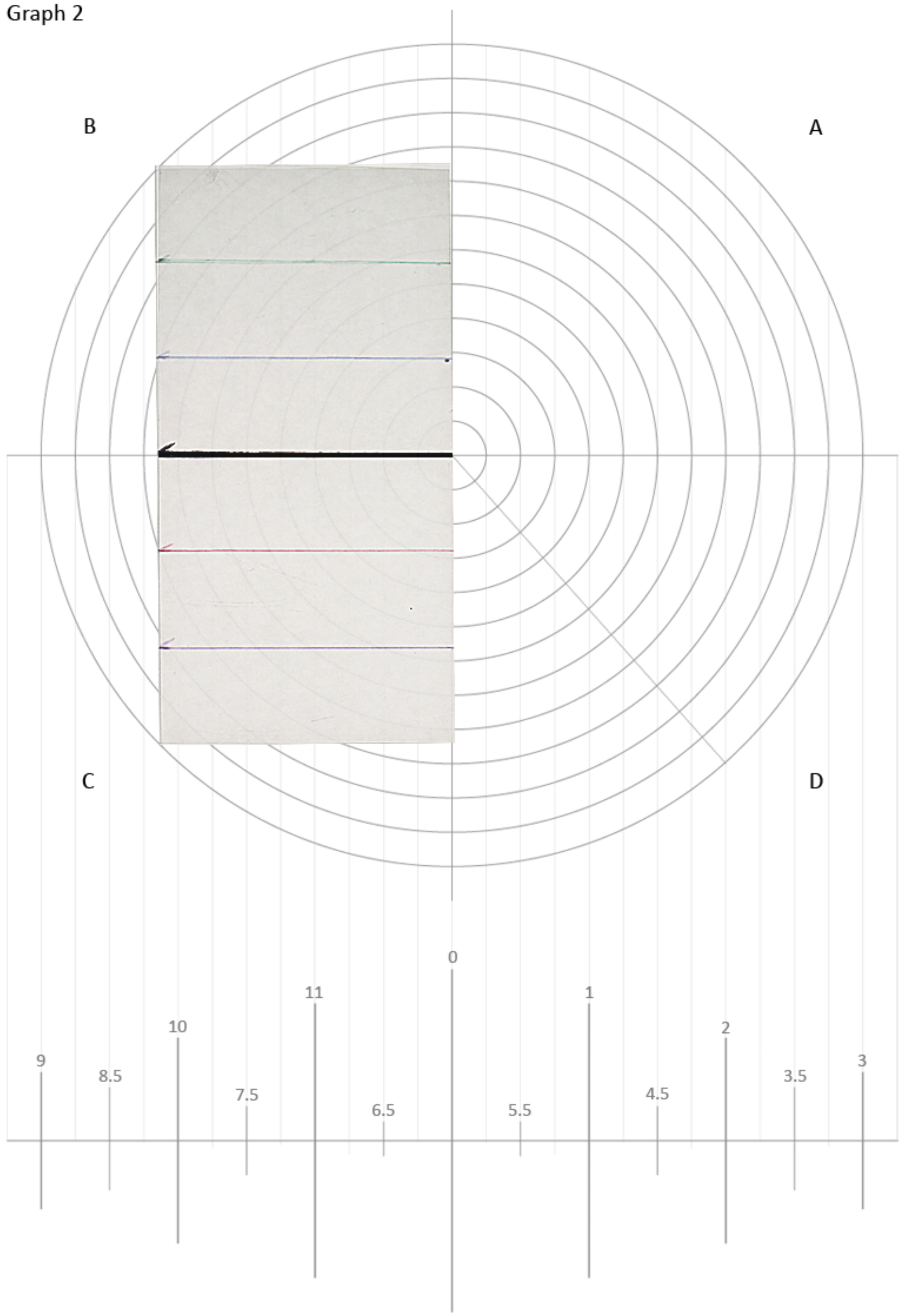
[95] Paper Folding Graph Timeline by author.

Graph 1



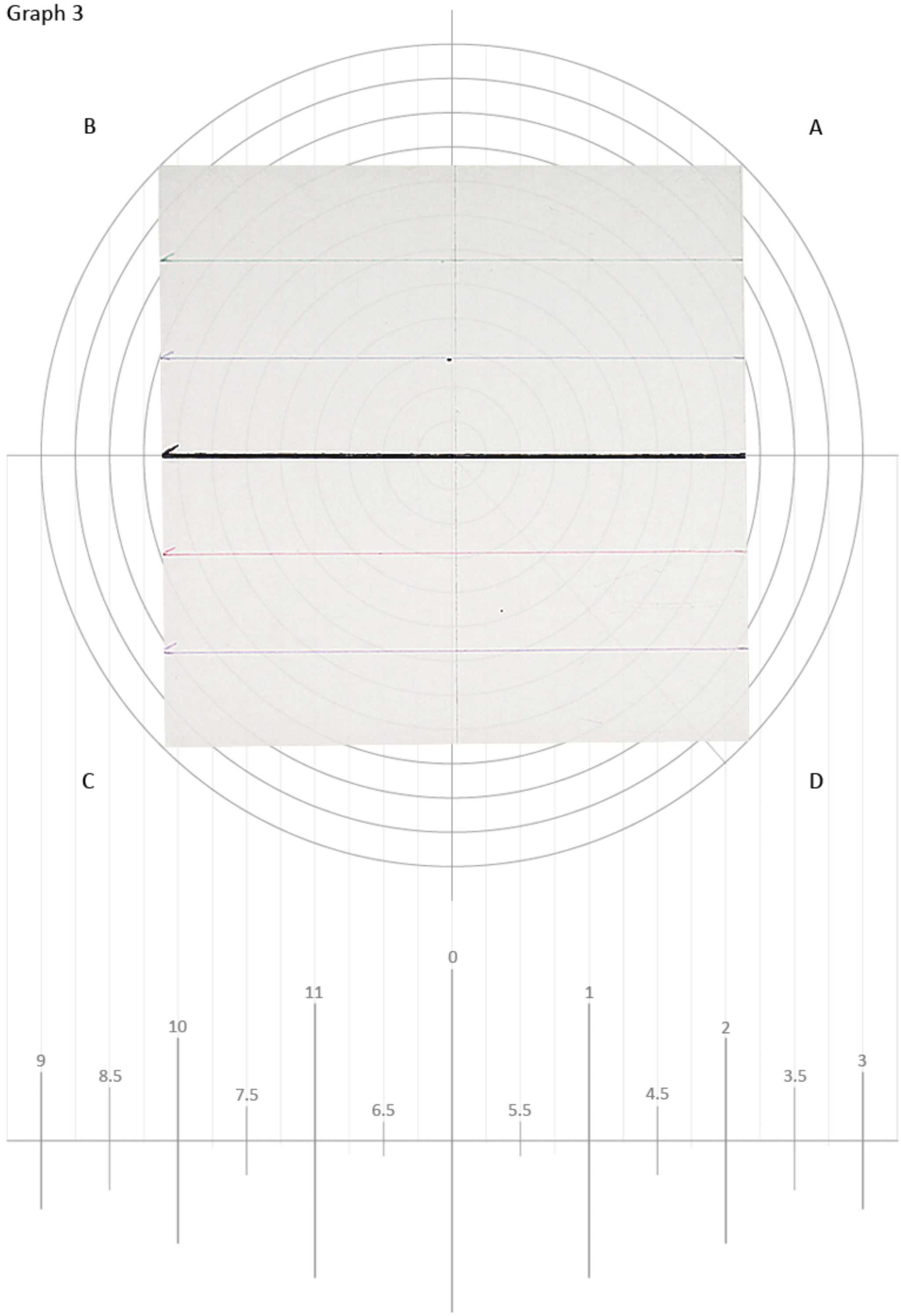
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

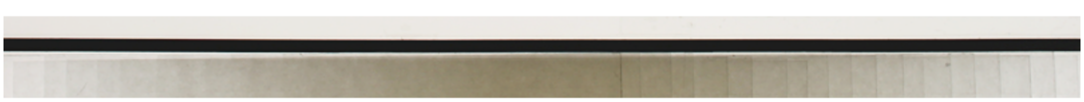
Graph 2



NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

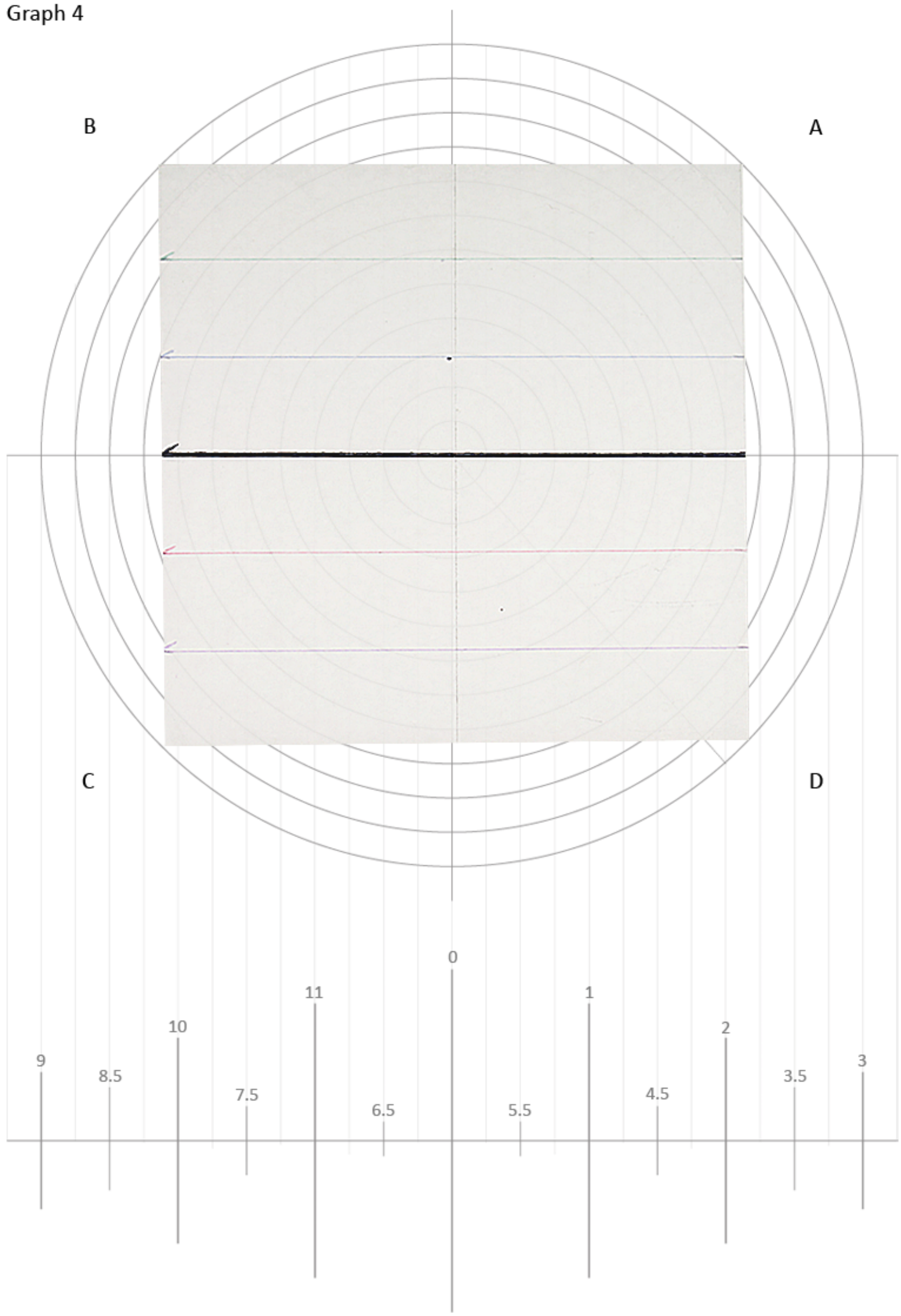
Graph 3





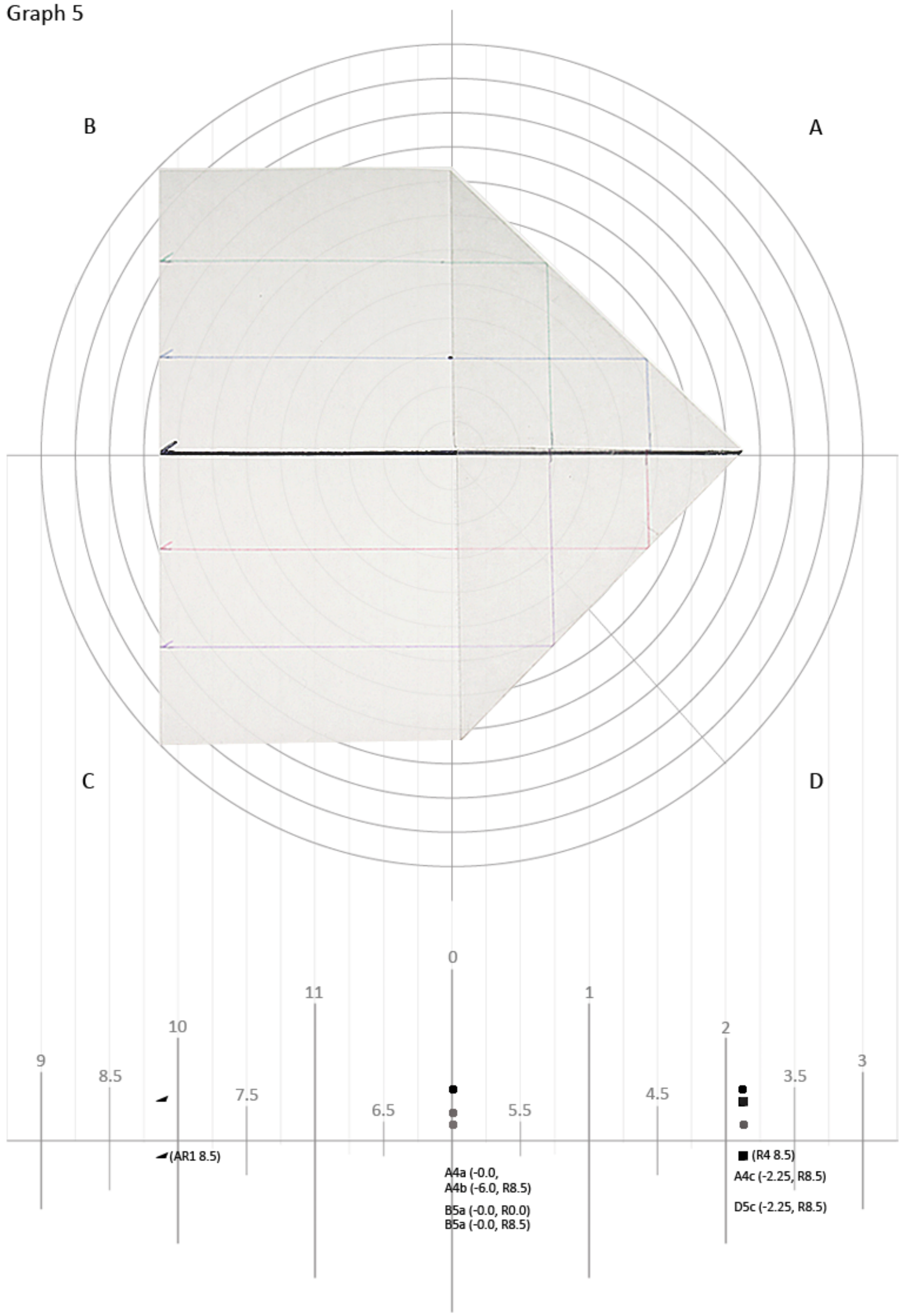
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 4



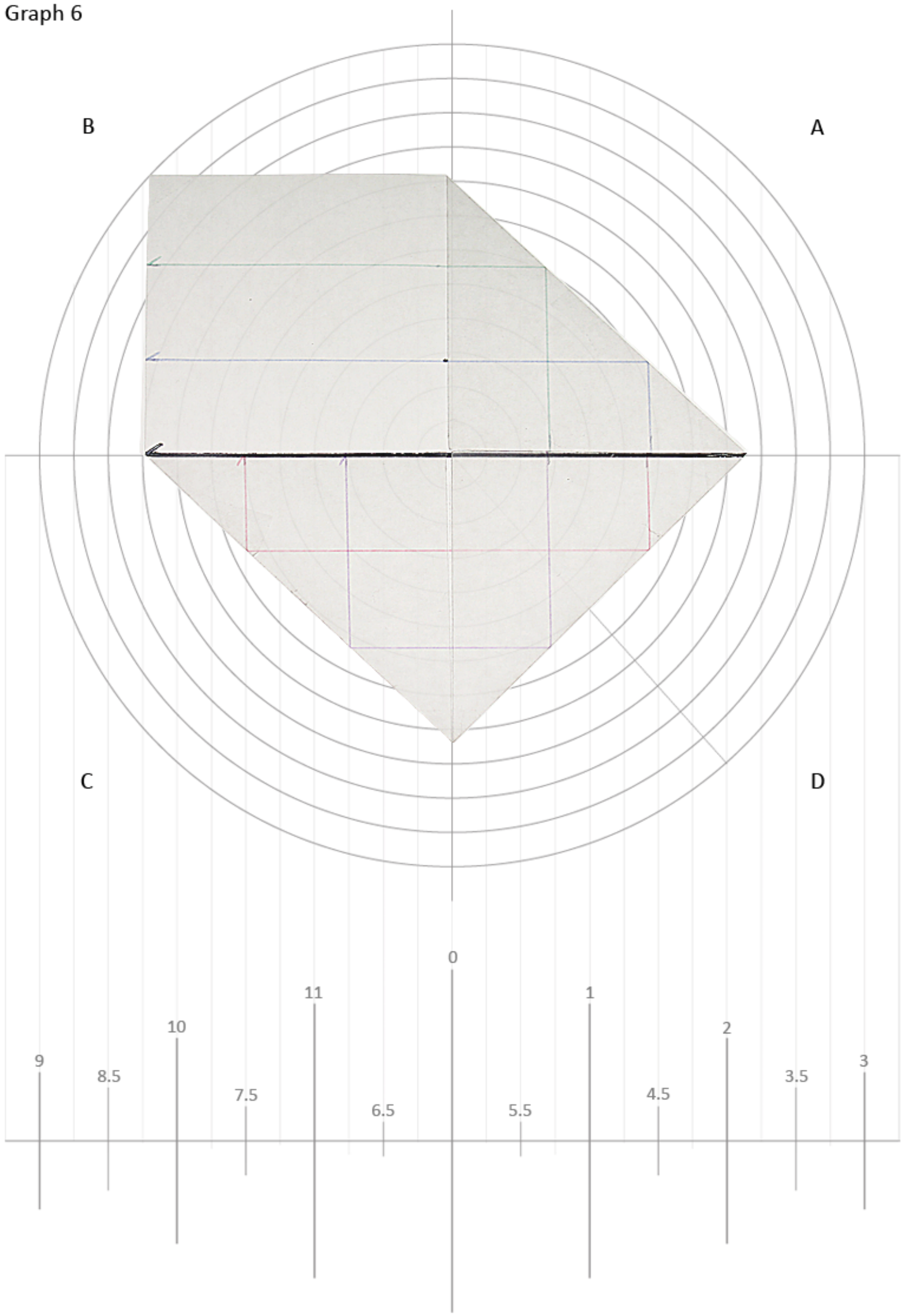
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 5



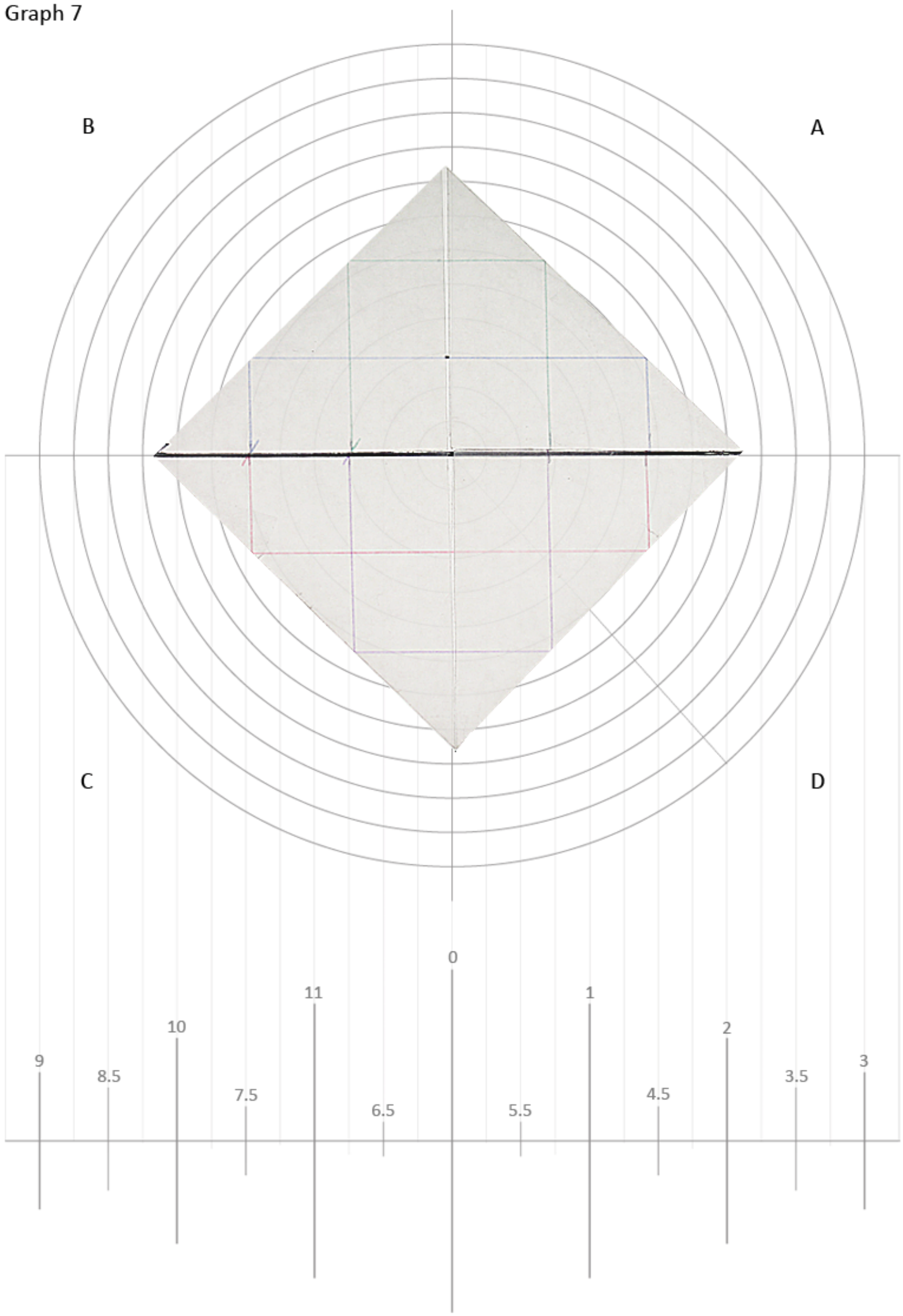
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 6



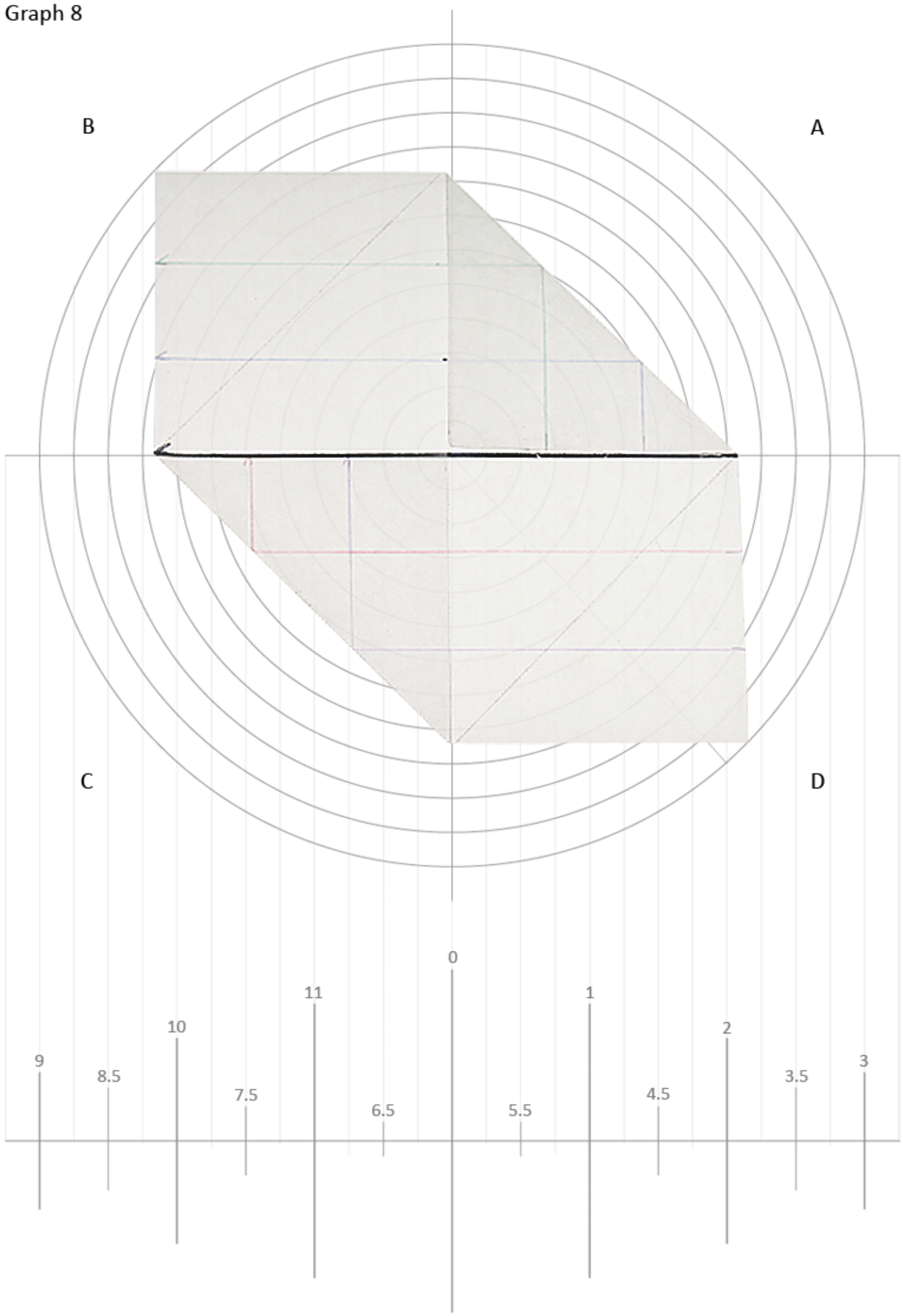
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

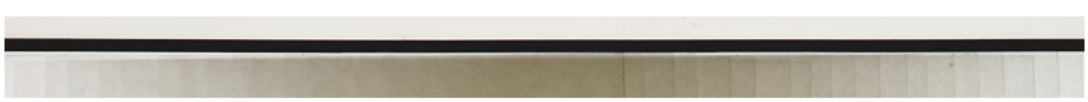
Graph 7



NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

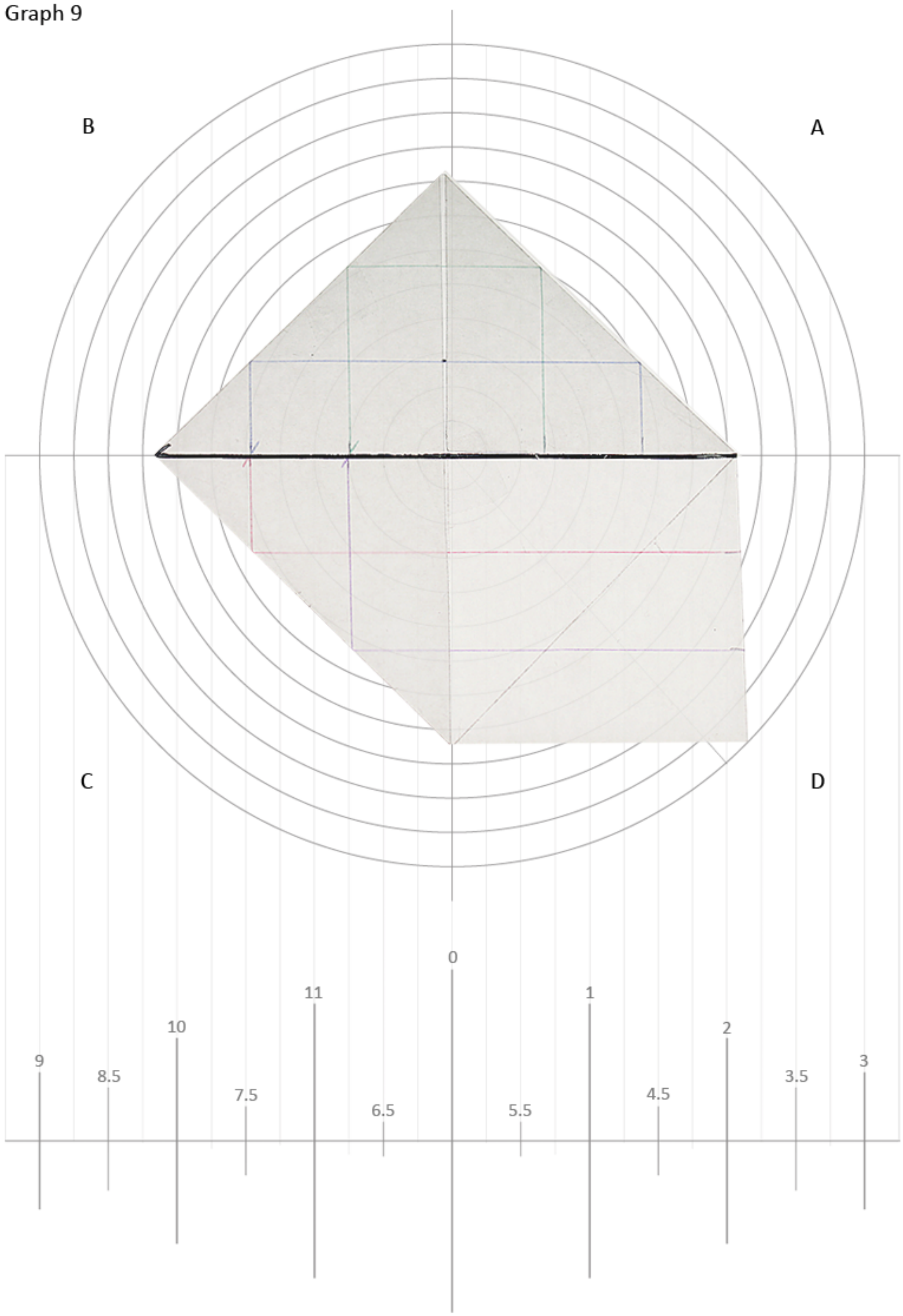
Graph 8





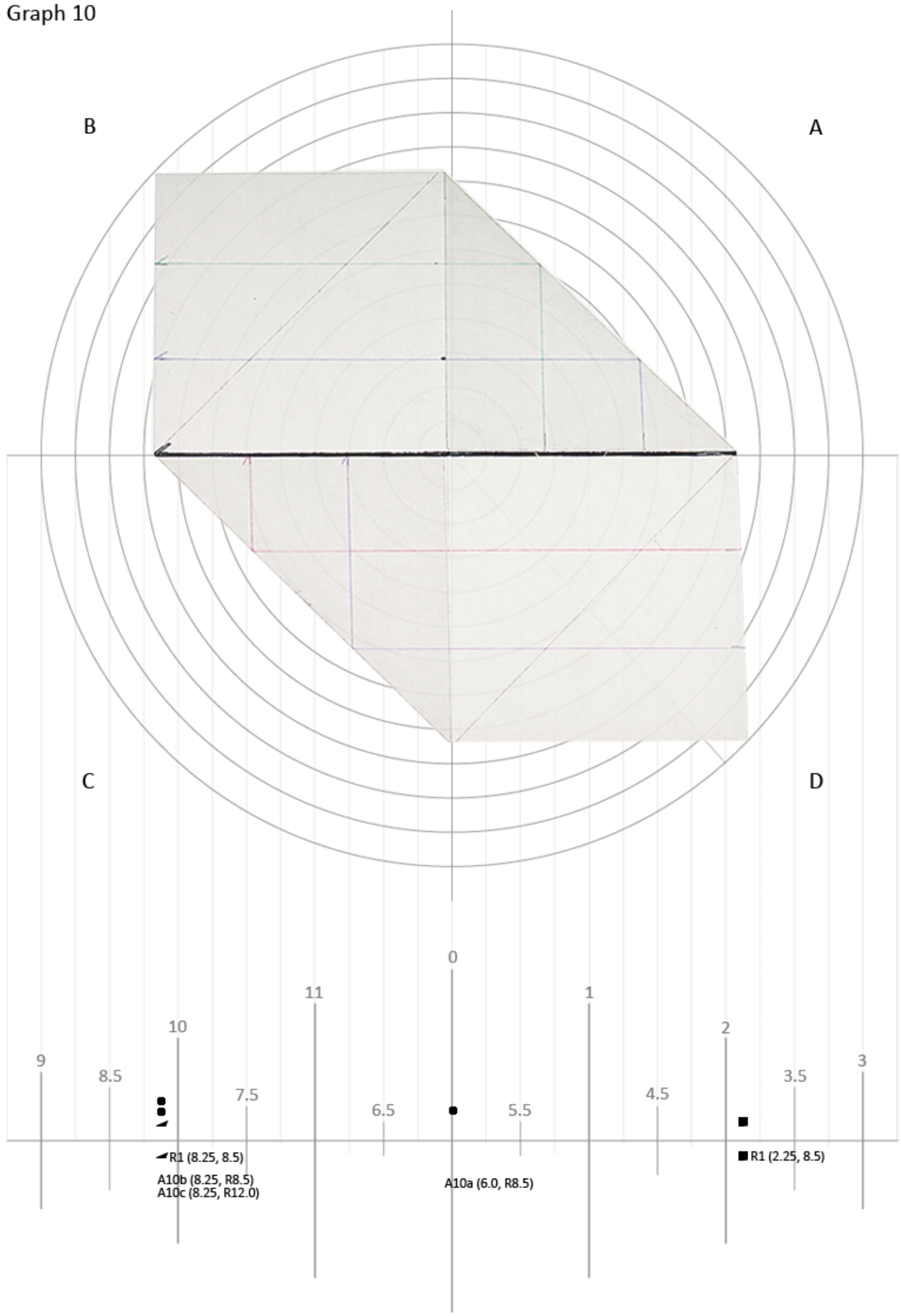
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

Graph 9



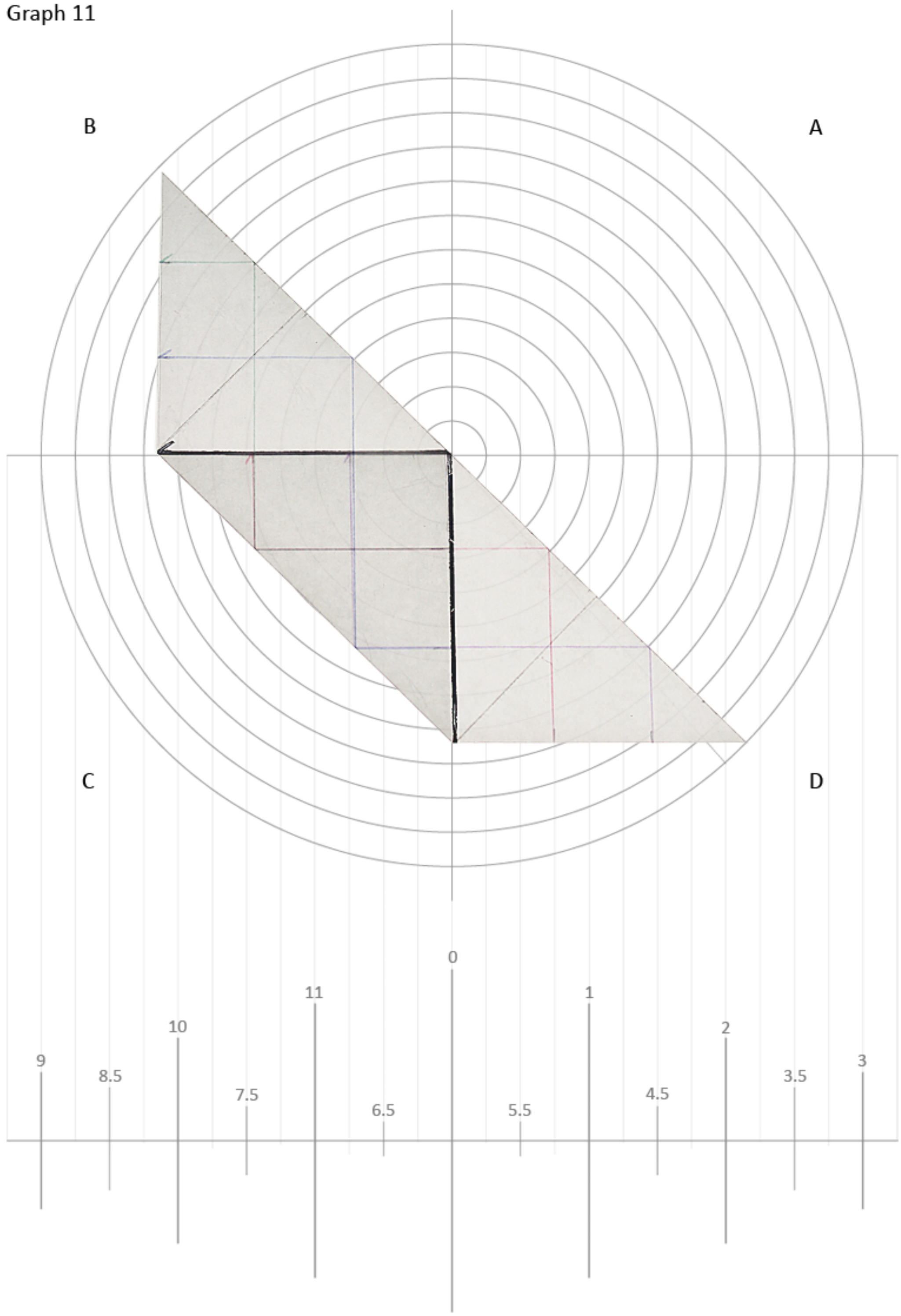
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 10



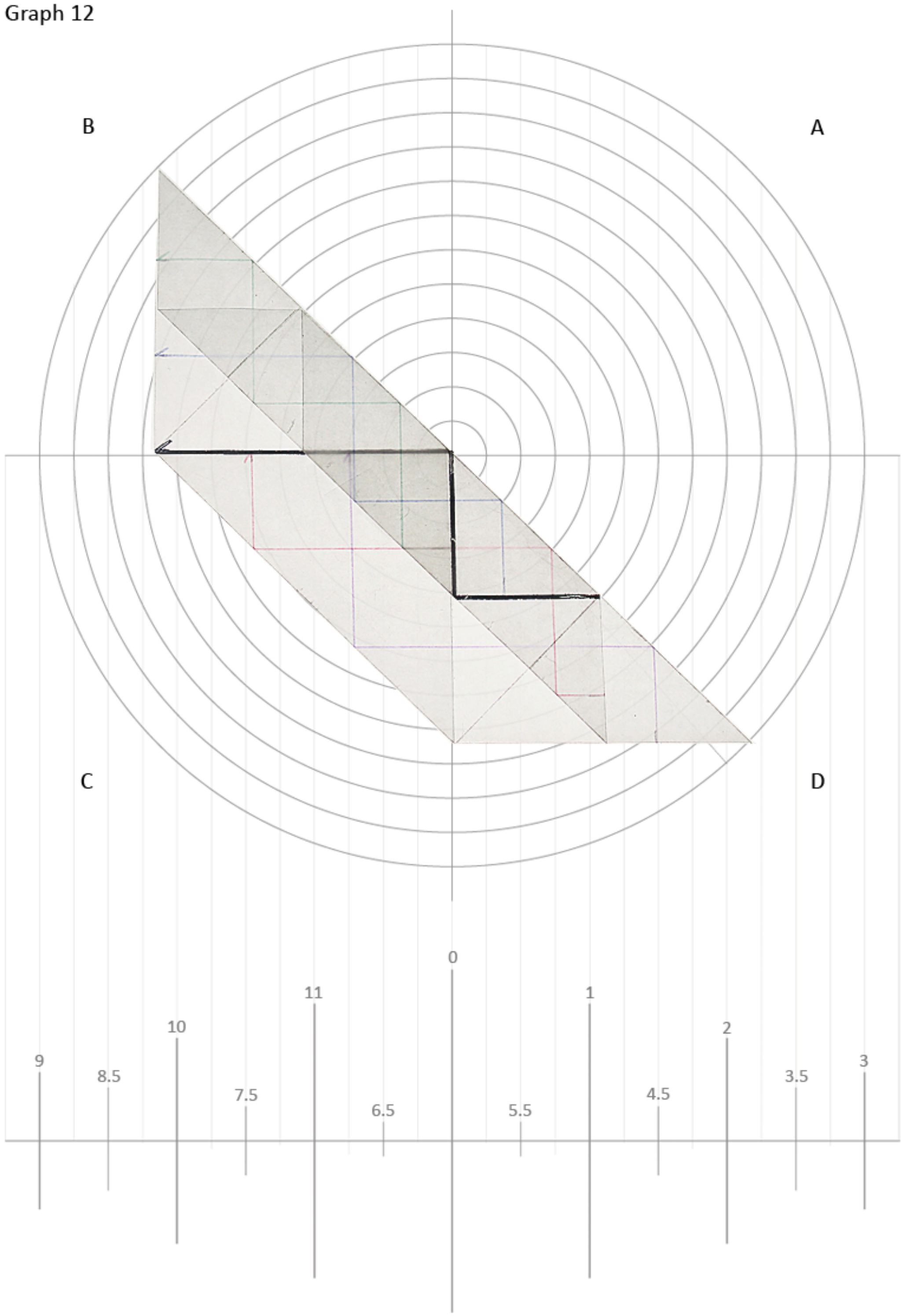
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

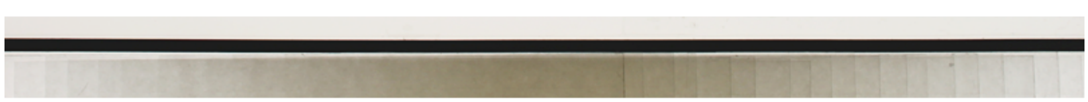
Graph 11



NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

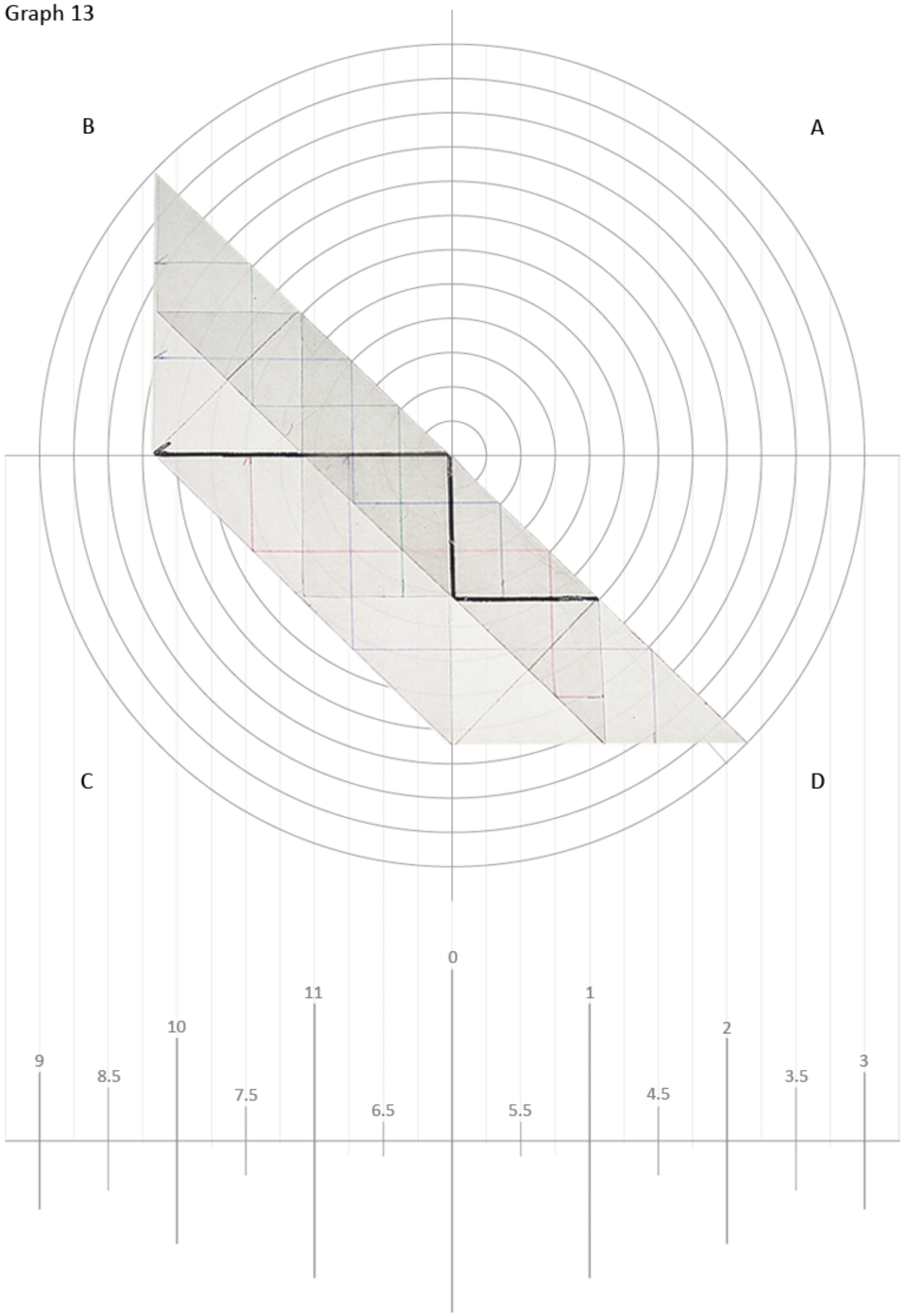
Graph 12

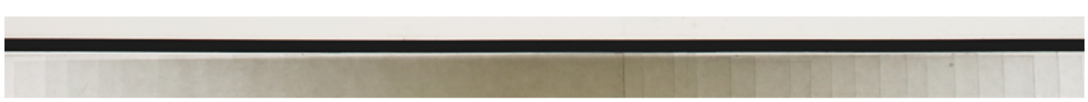




NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

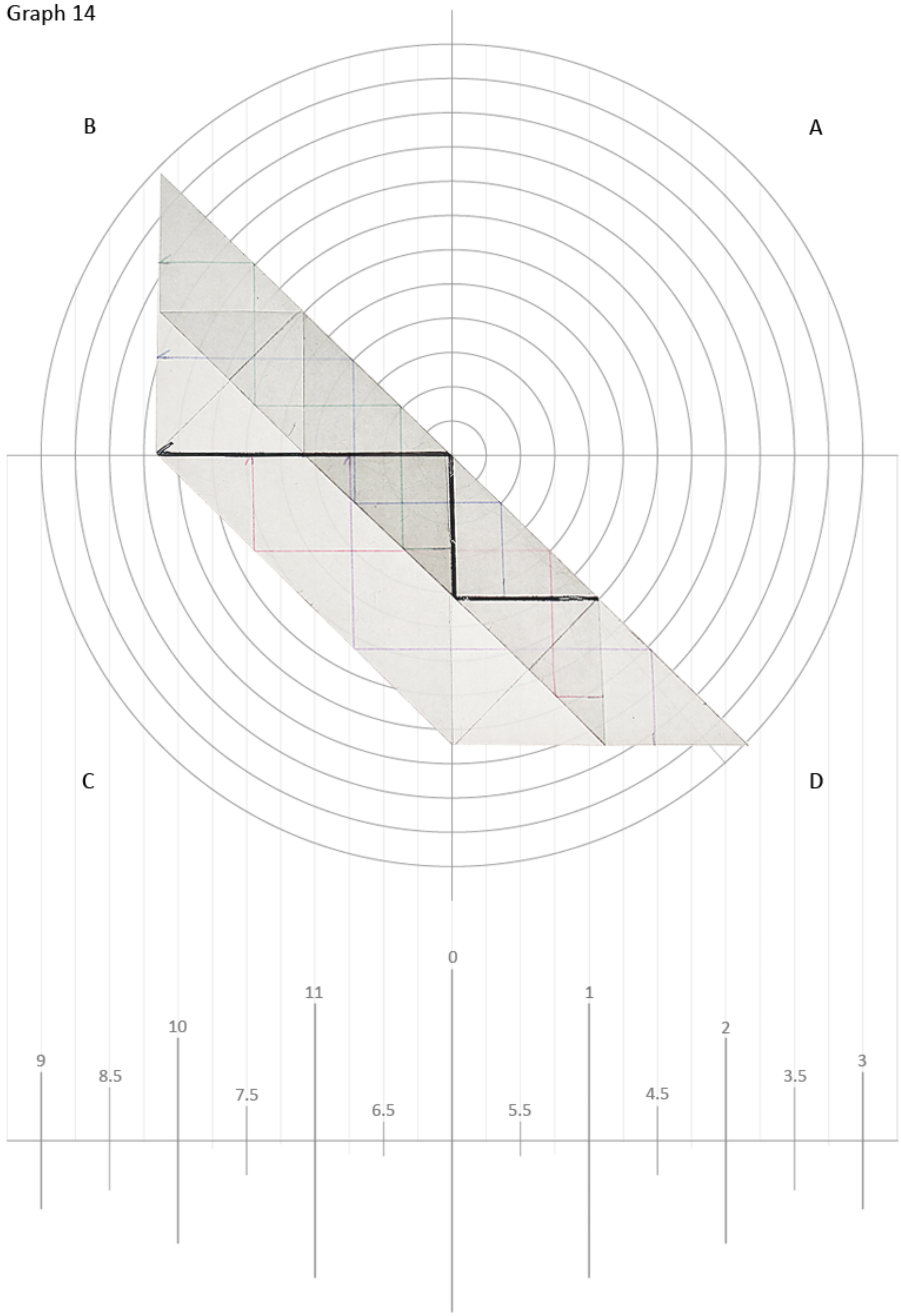
Graph 13





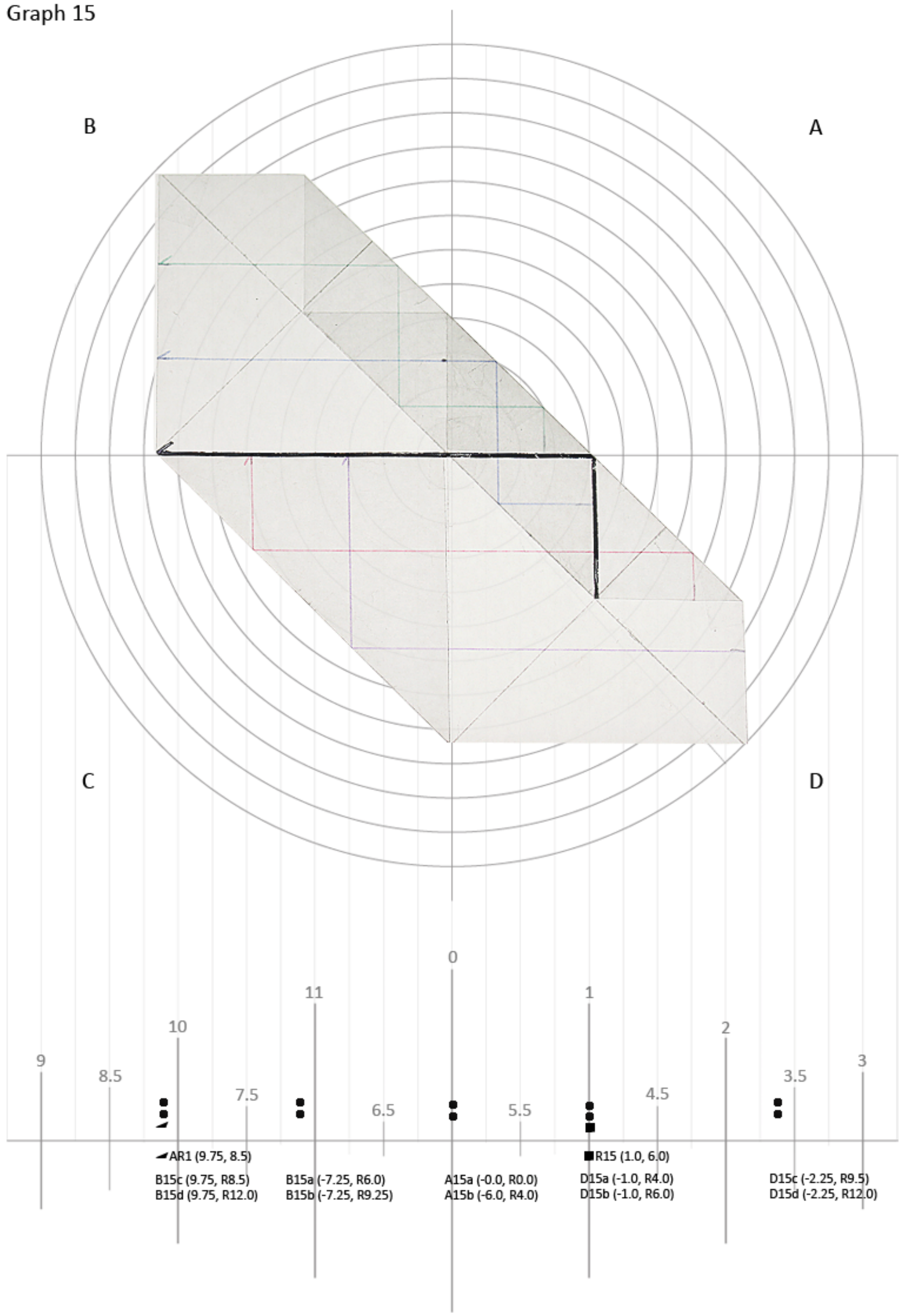
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

Graph 14



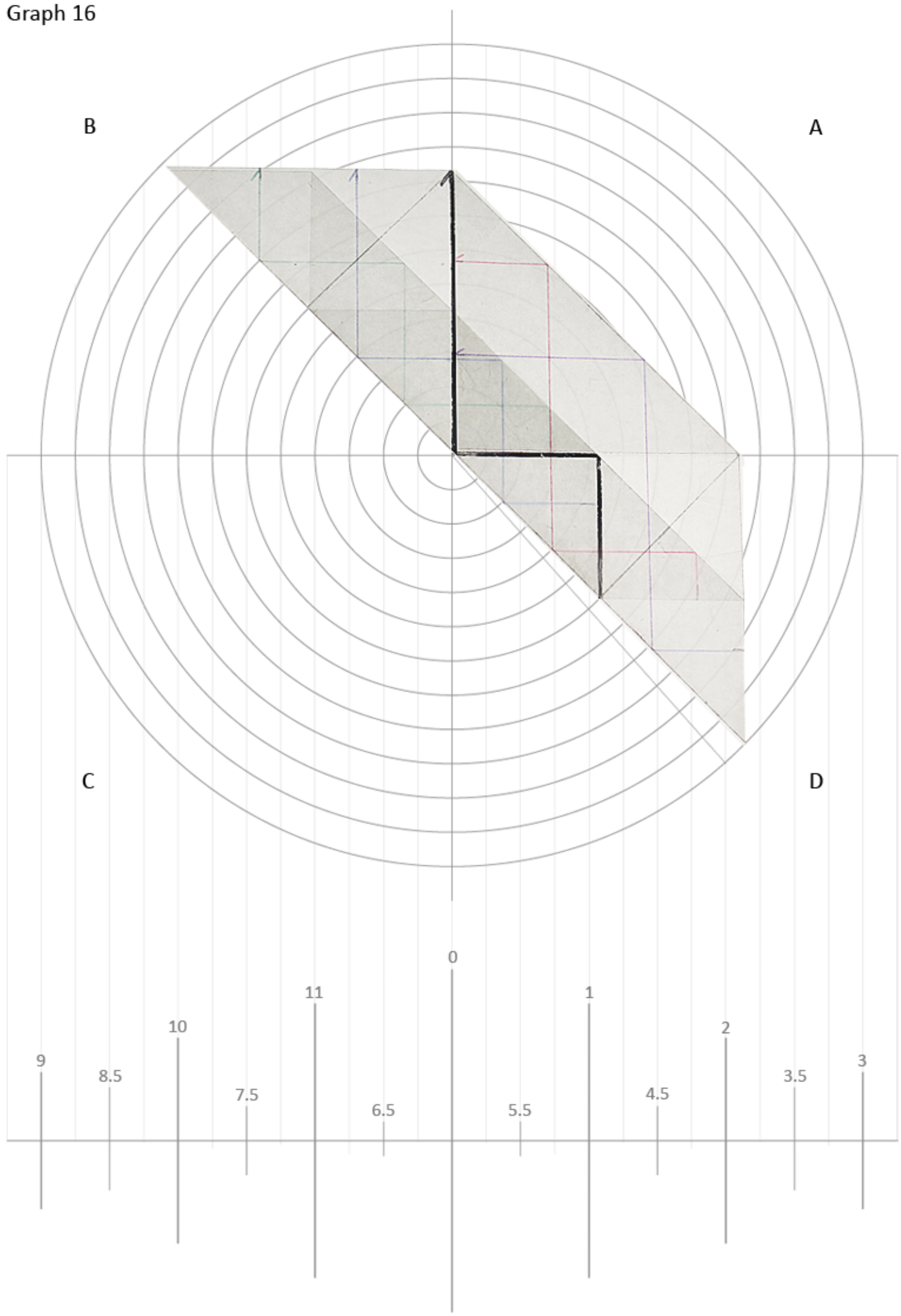
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 15



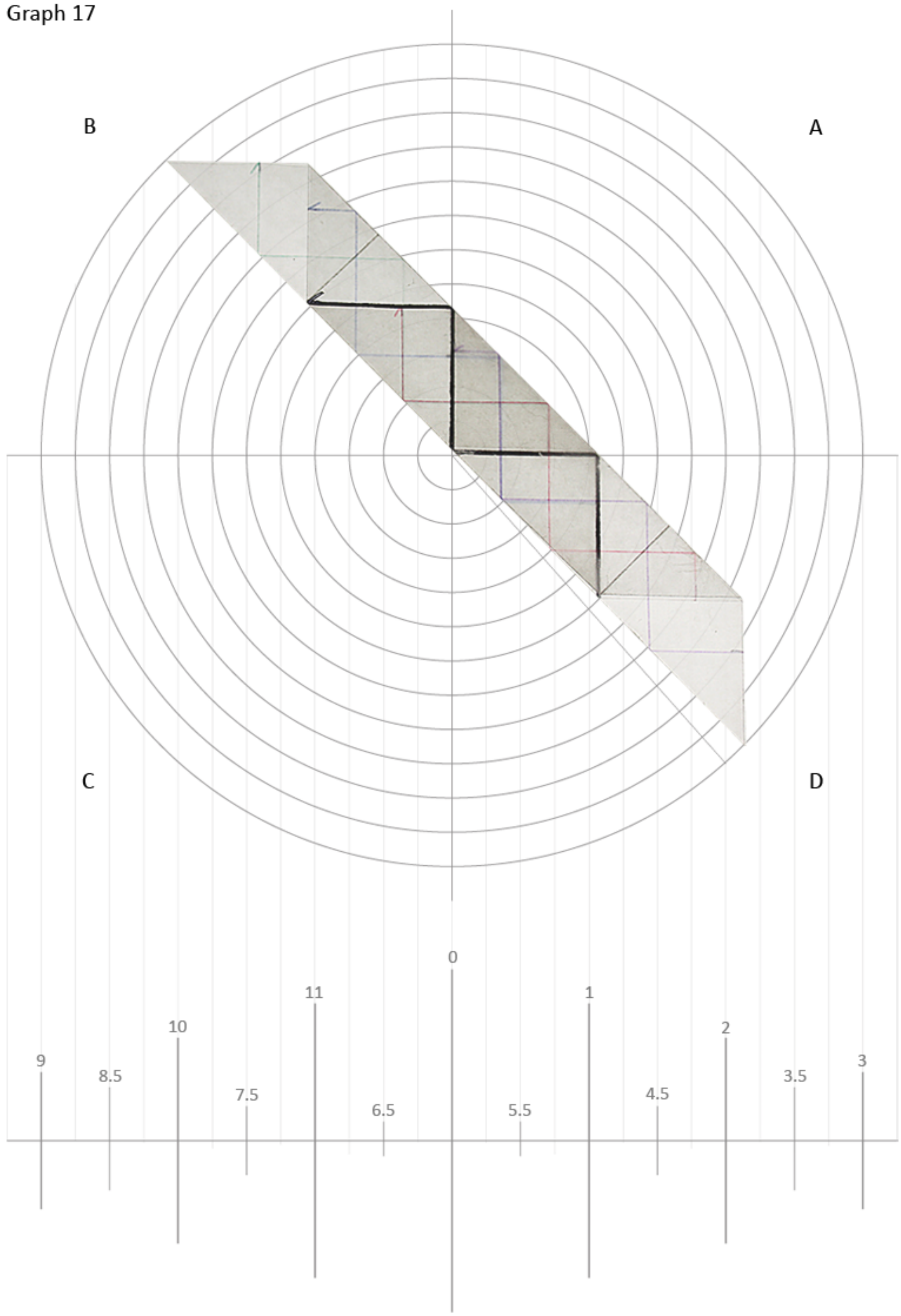
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 16



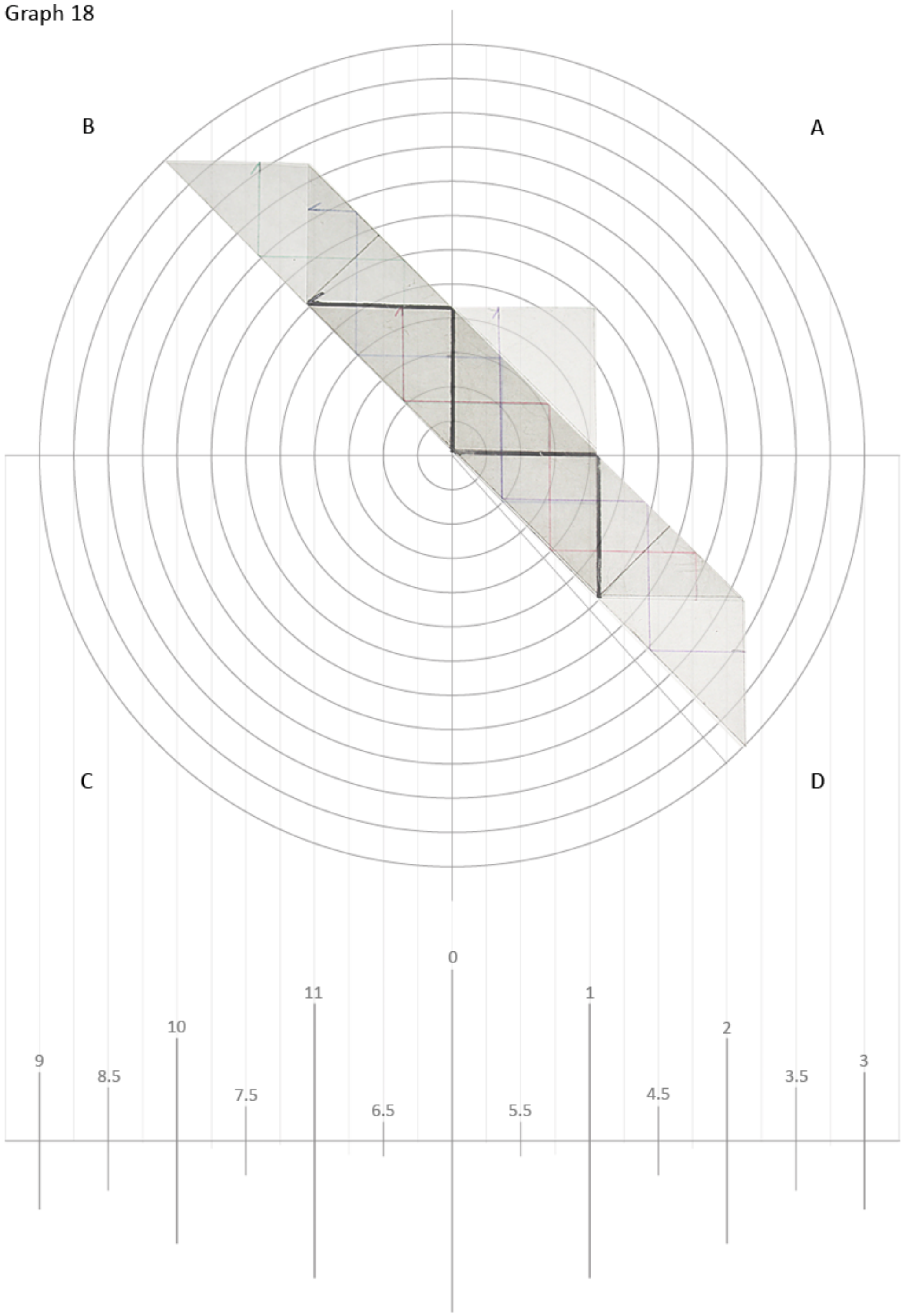
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

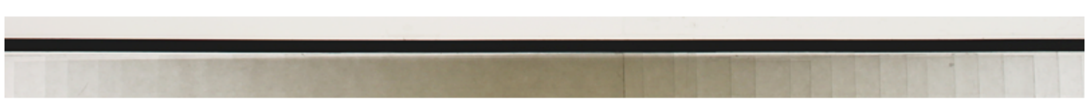
Graph 17



NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

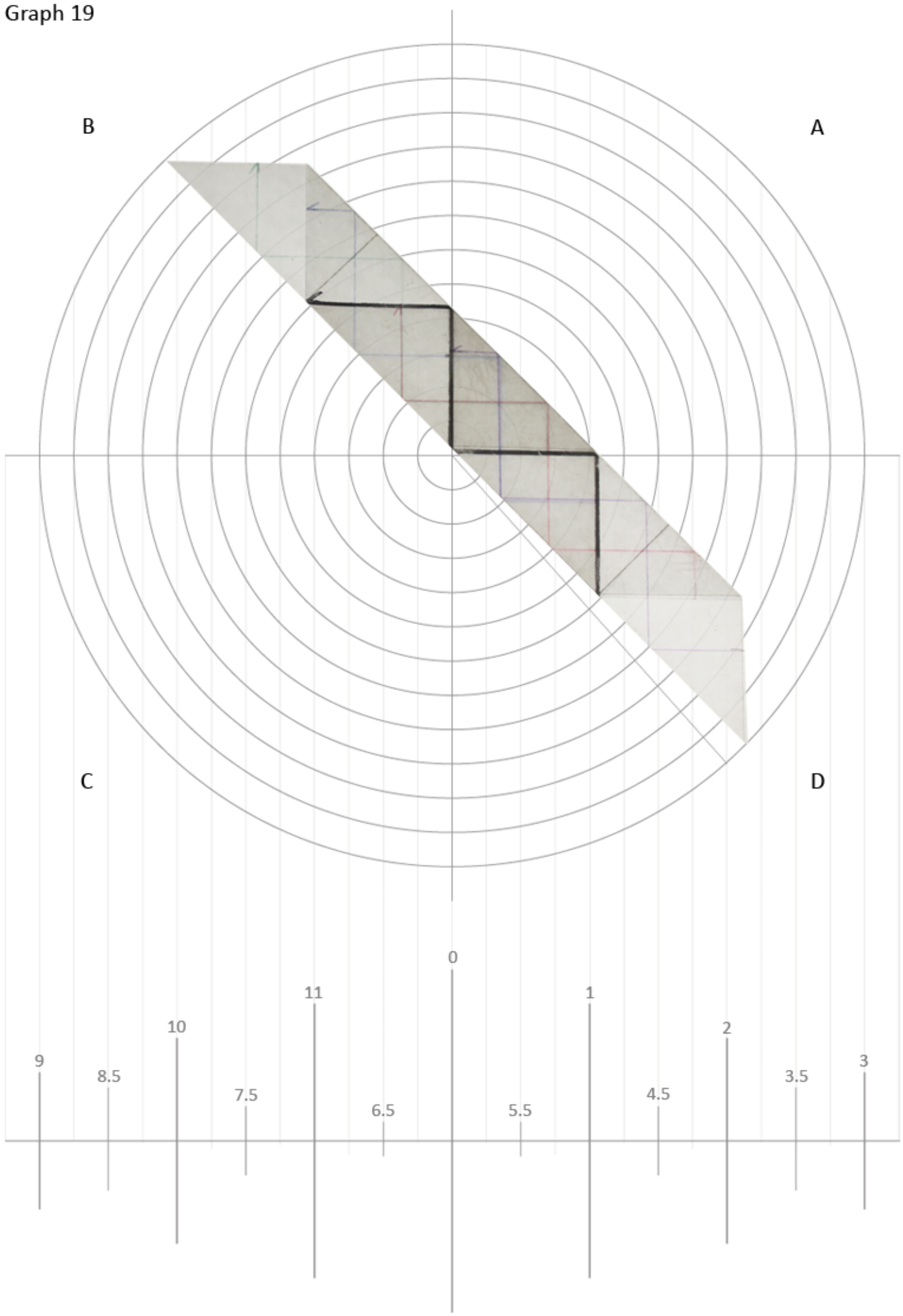
Graph 18





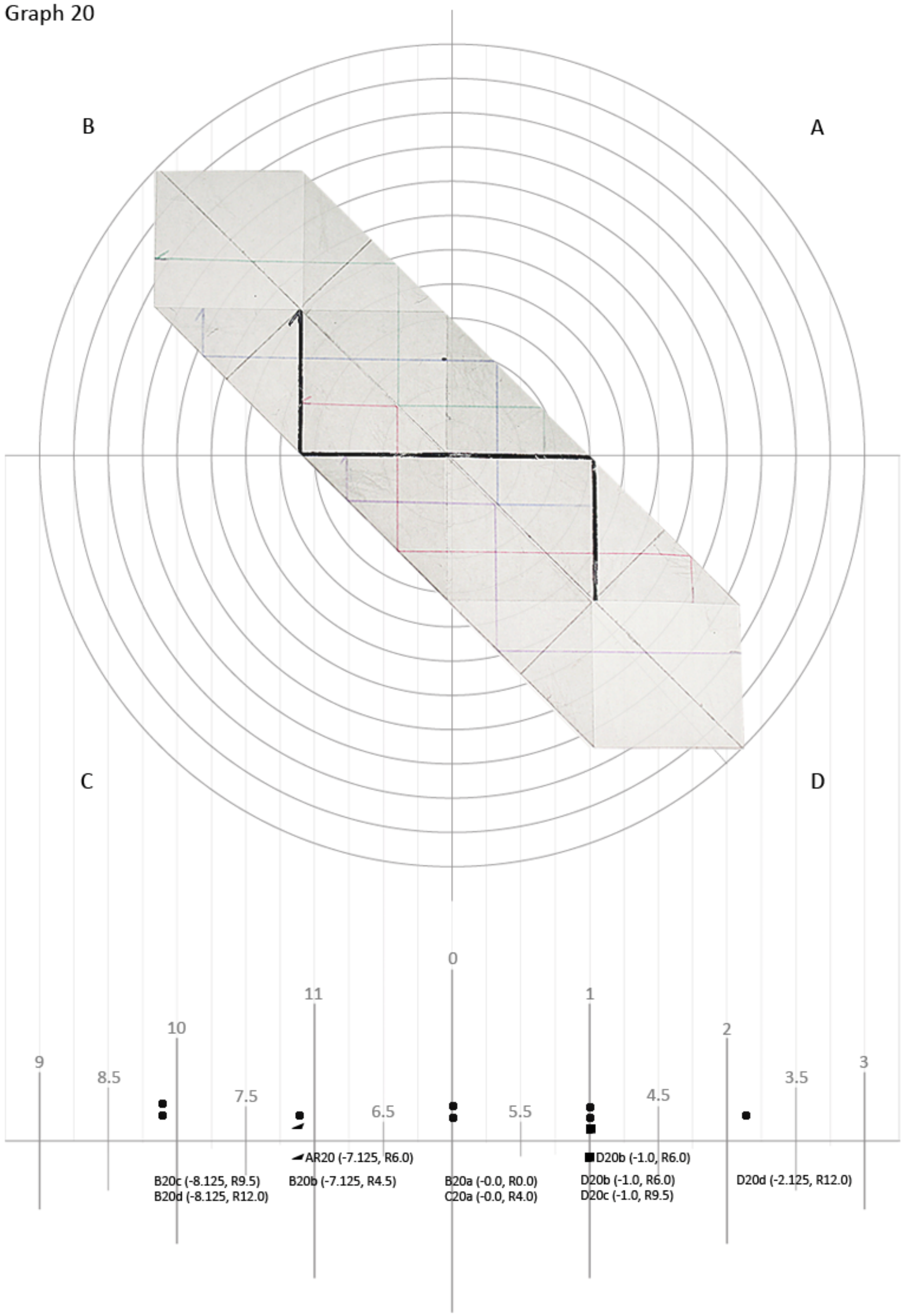
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

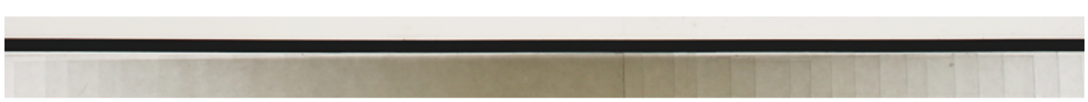
Graph 19



NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

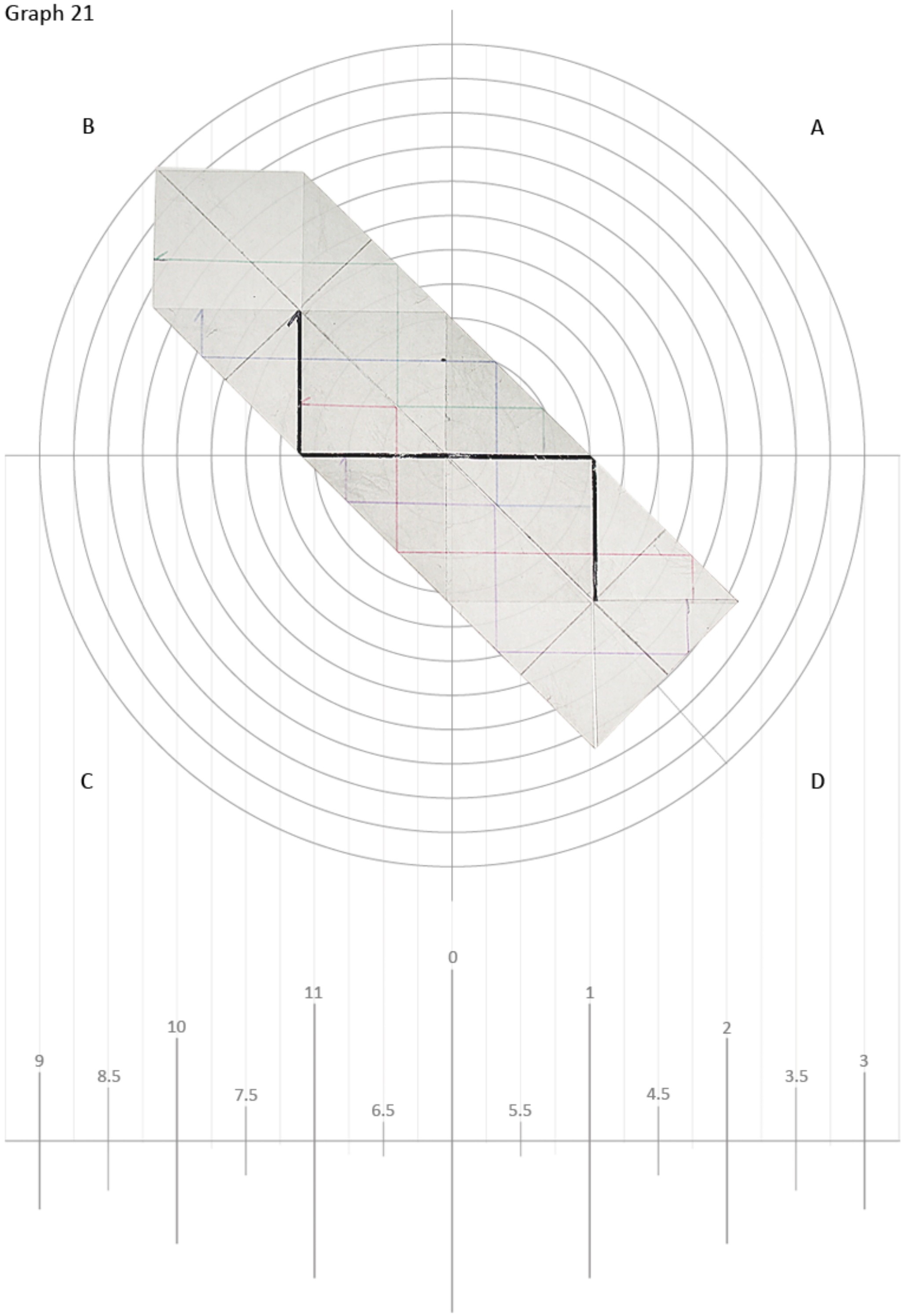
Graph 20





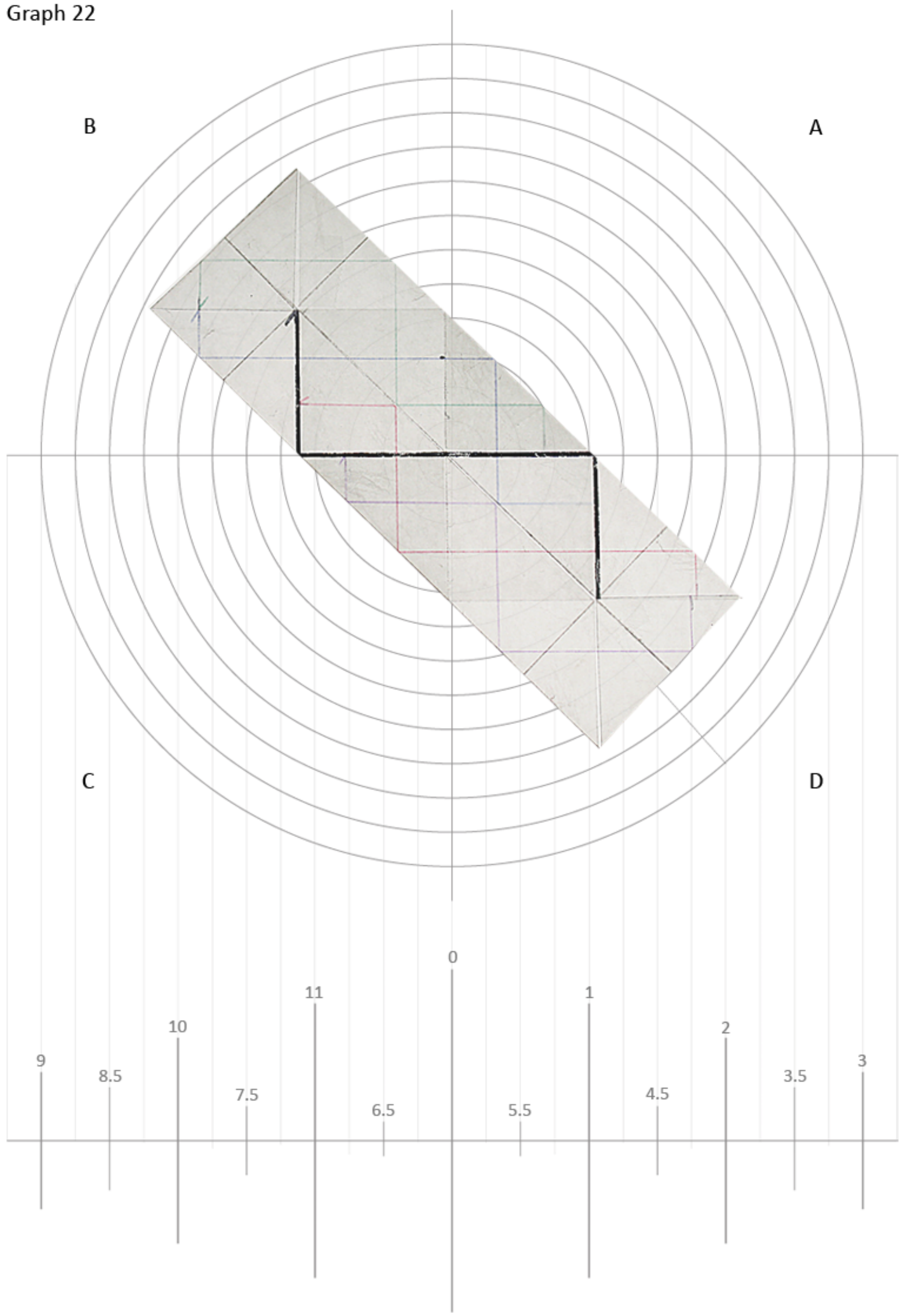
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 21



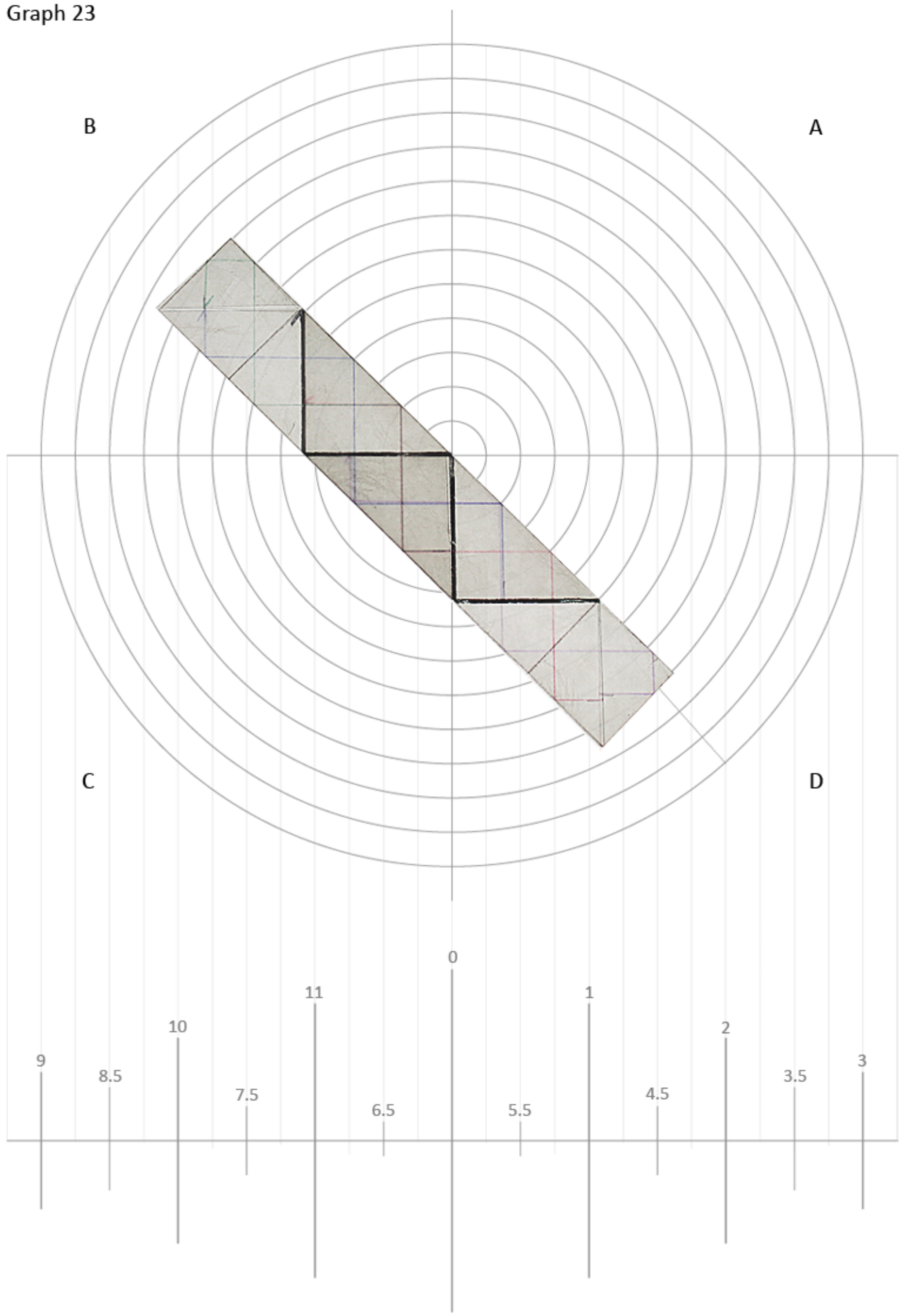
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 22



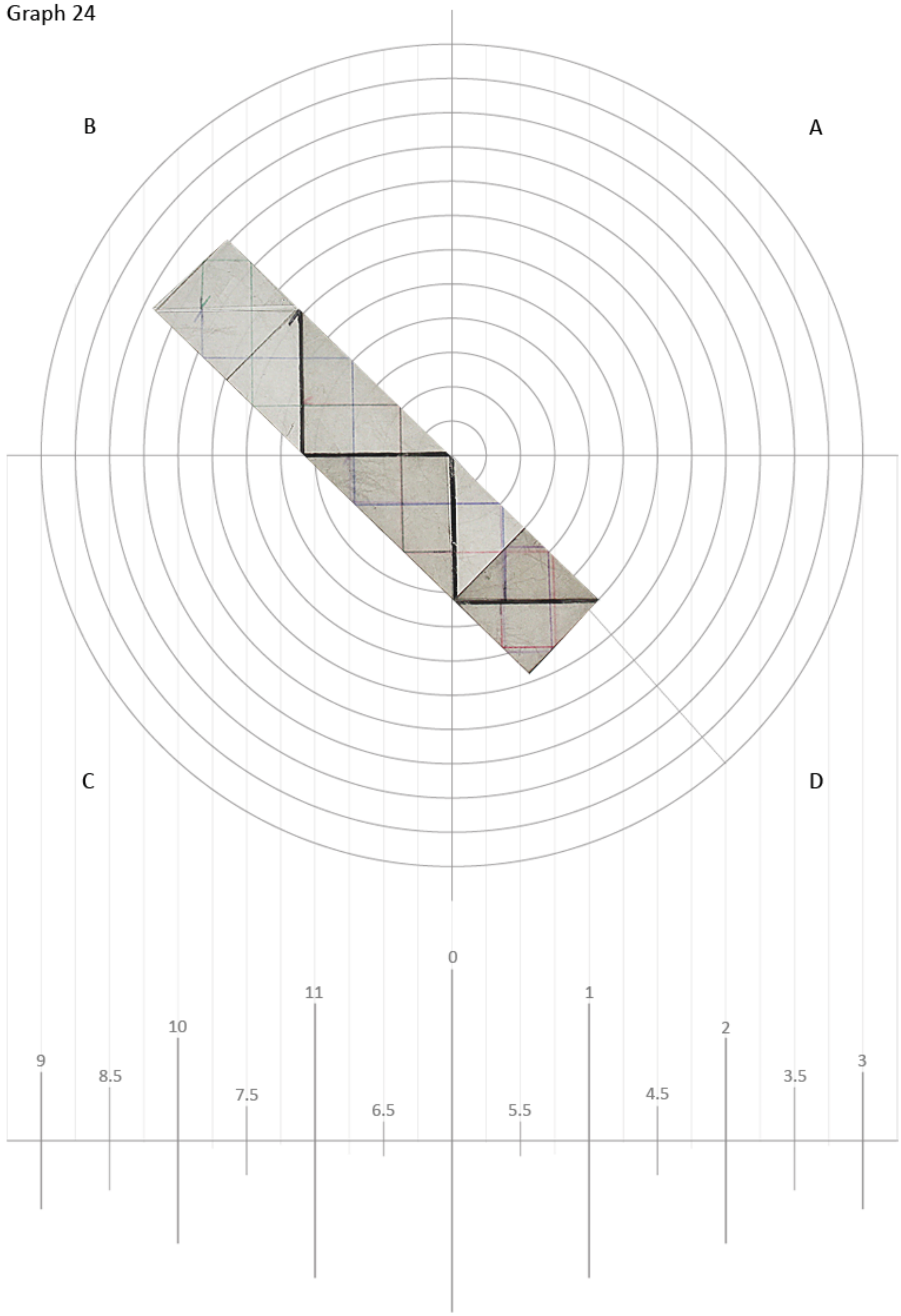
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 23



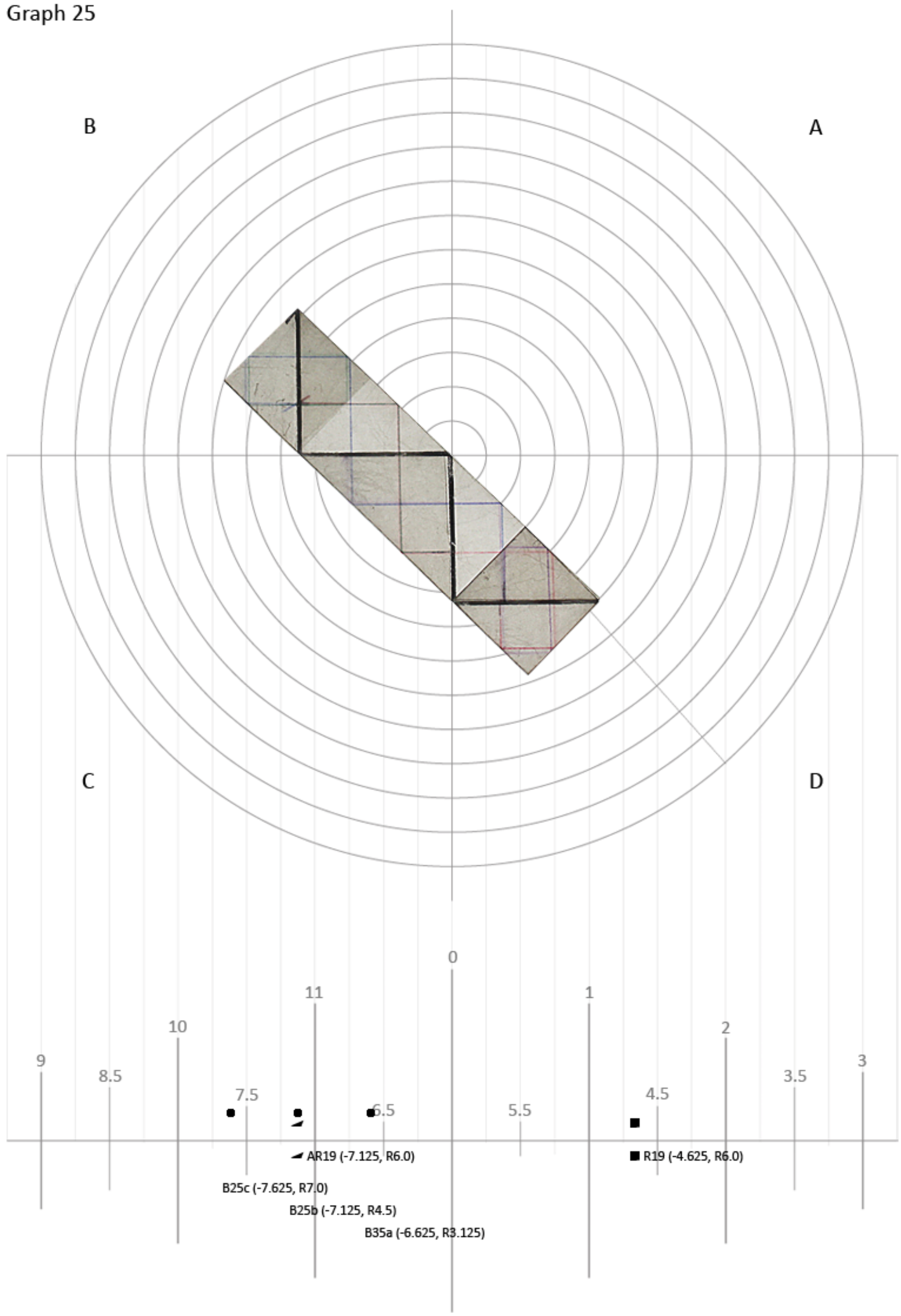
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 24



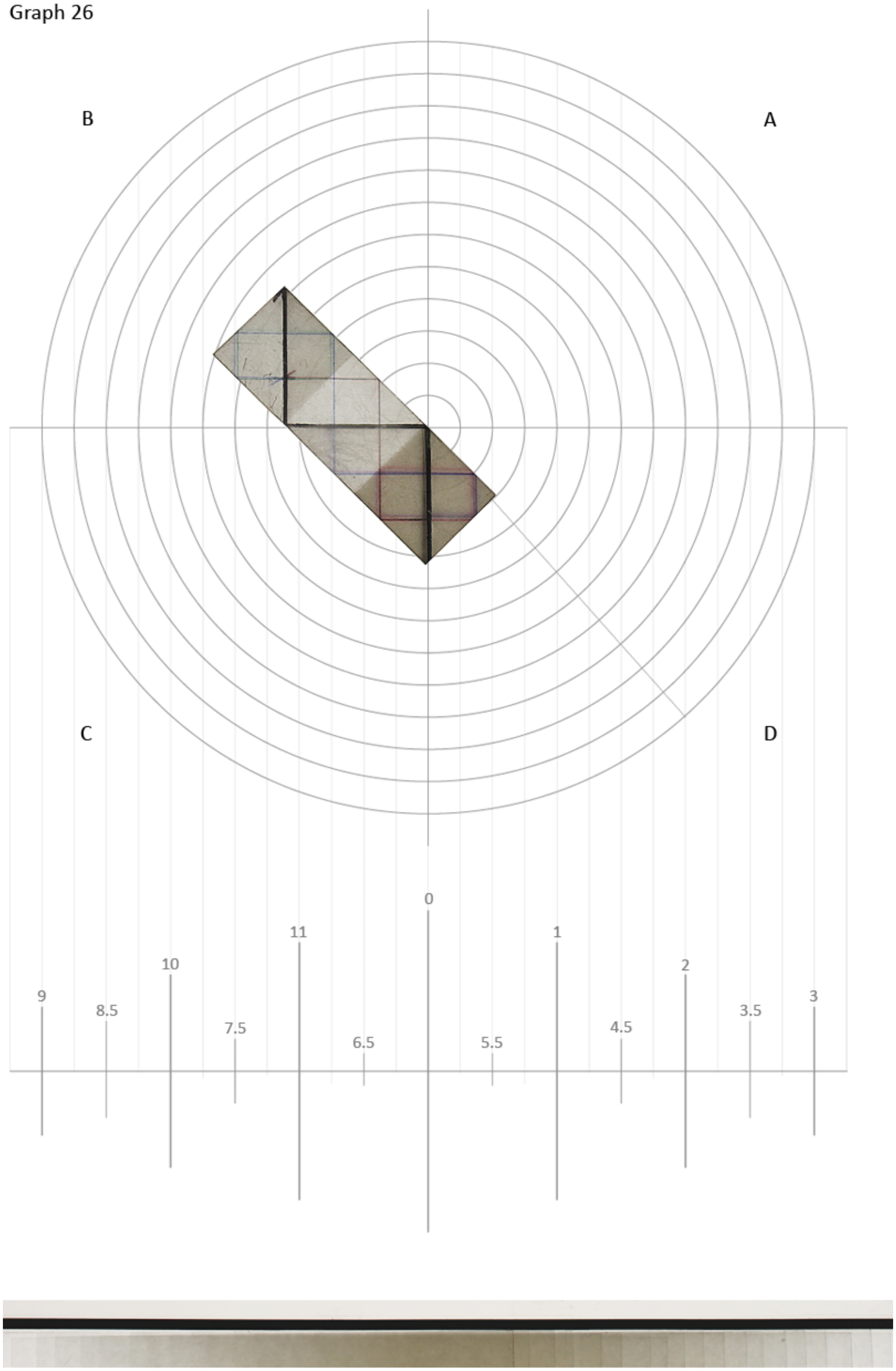
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

Graph 25



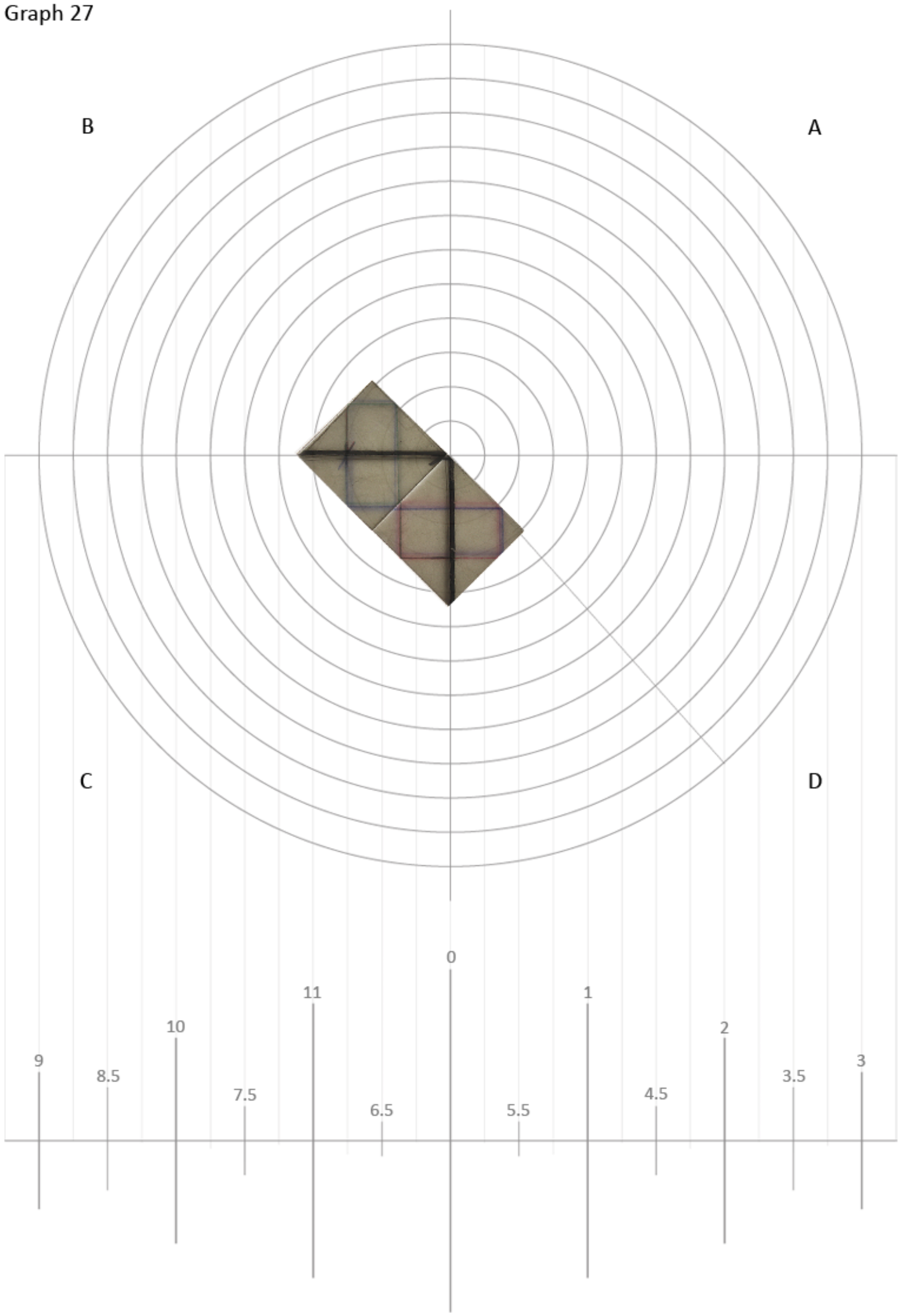
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 26



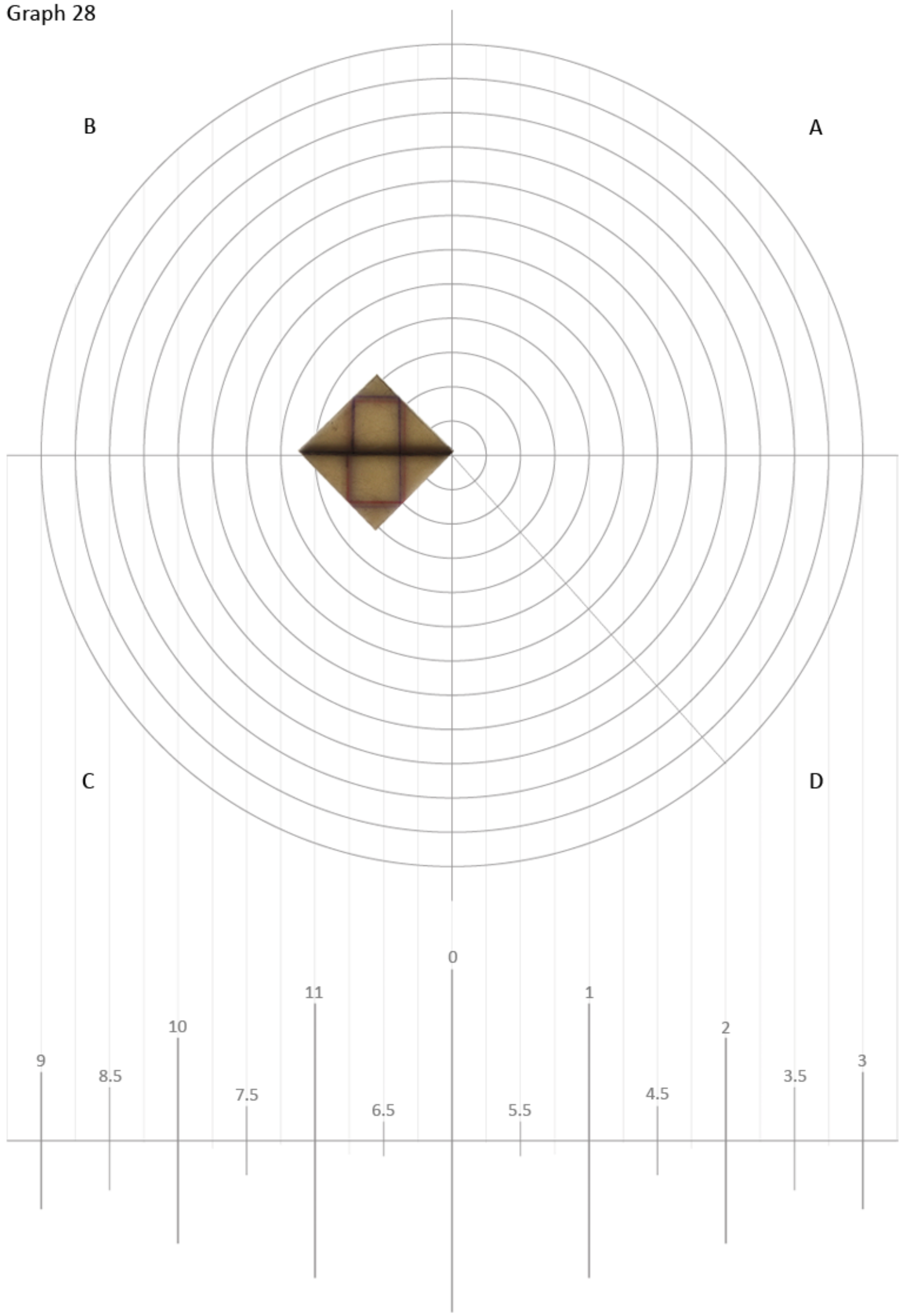
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 27



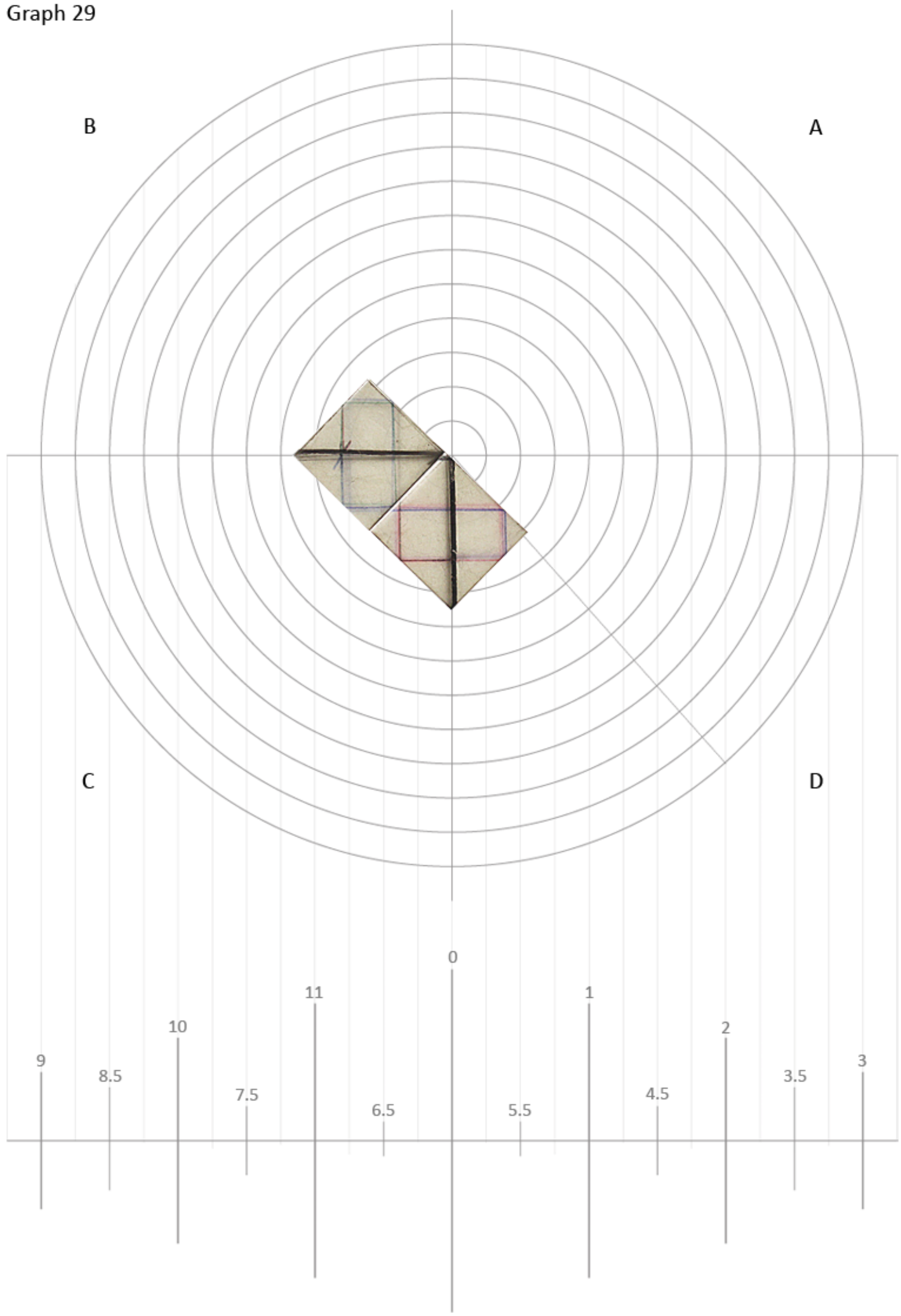
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

Graph 28



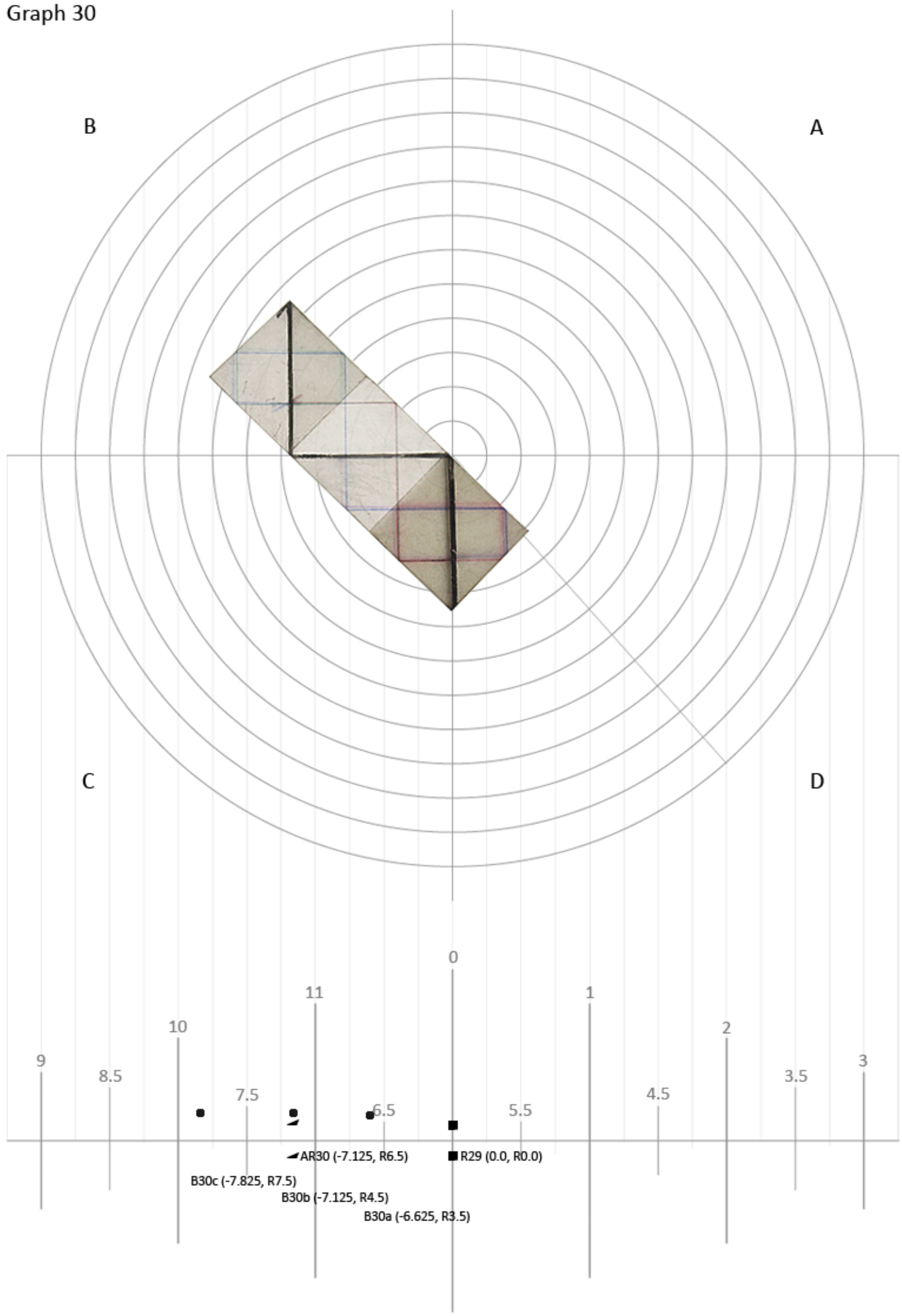
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 29



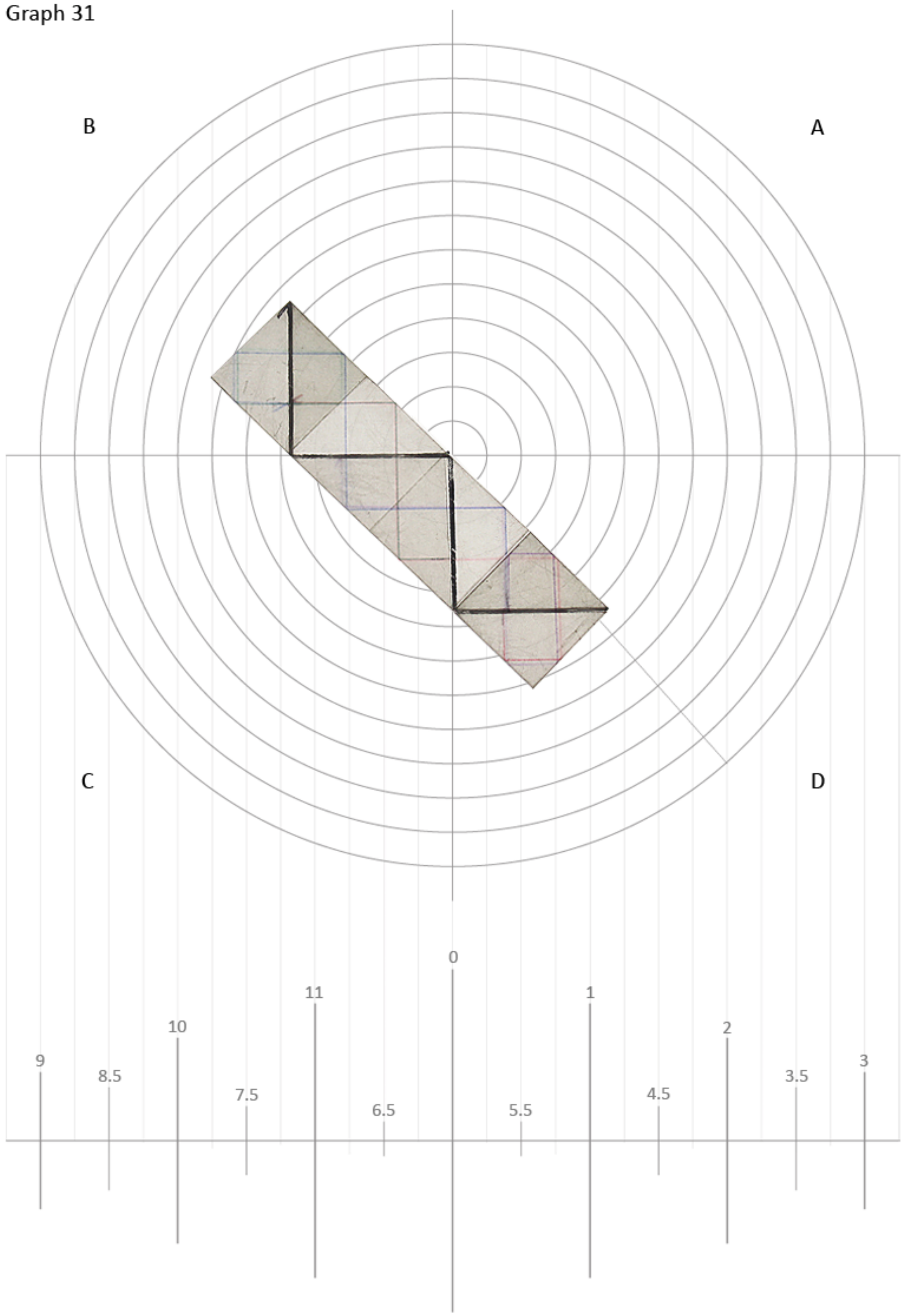
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 30



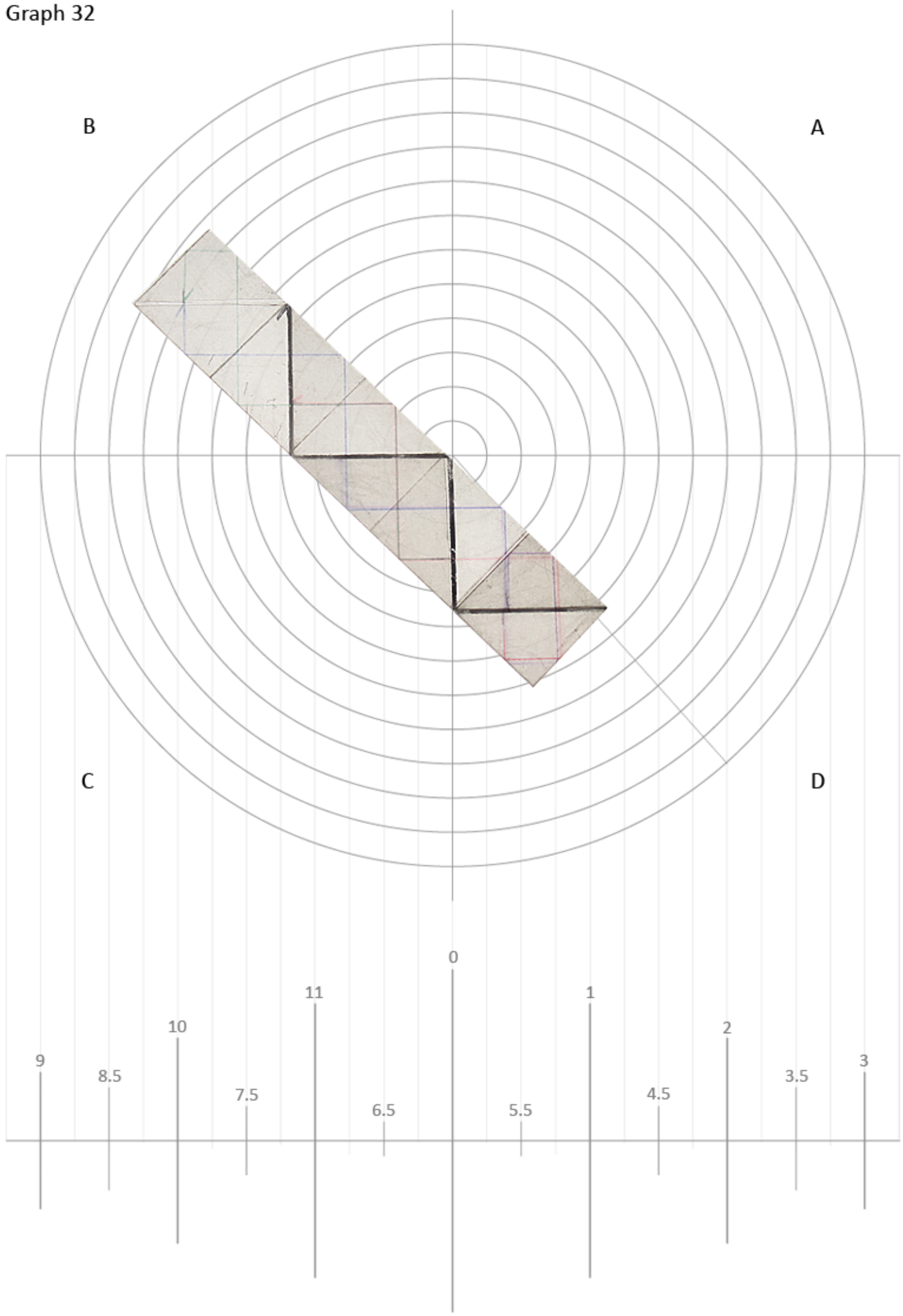
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

Graph 31



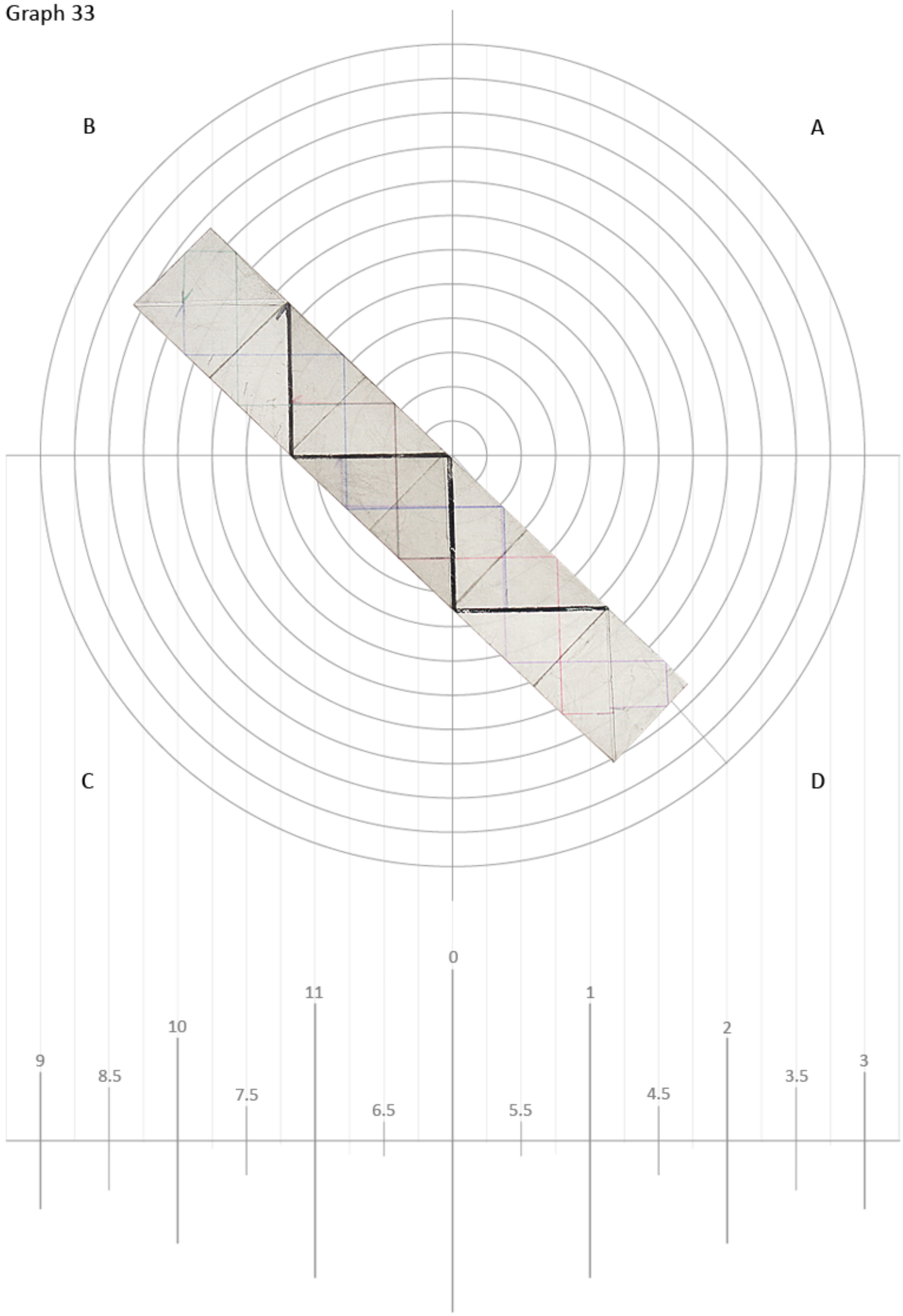
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

Graph 32



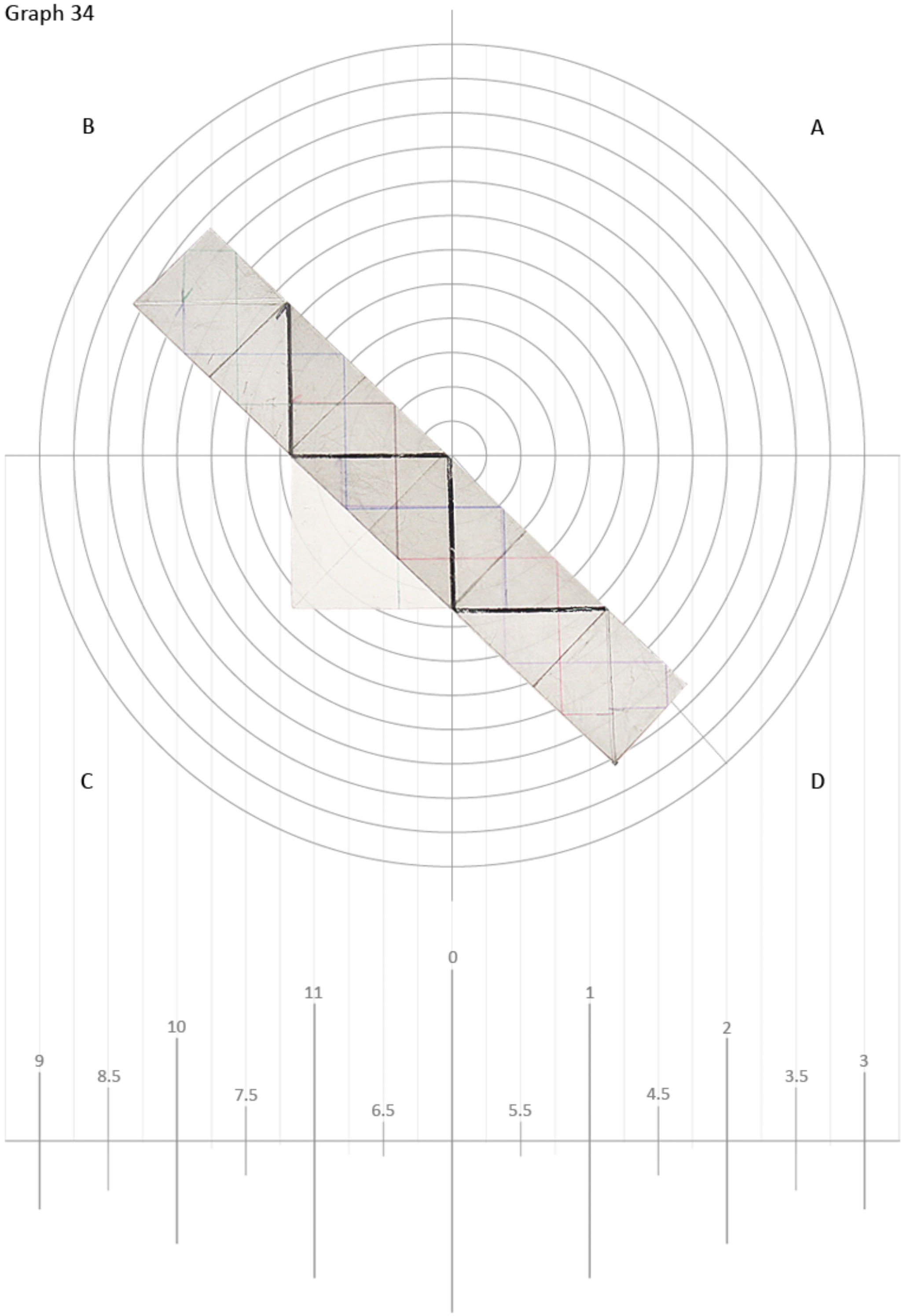
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

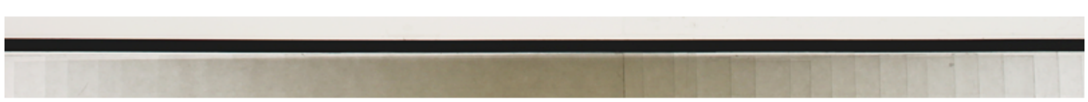
Graph 33



NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

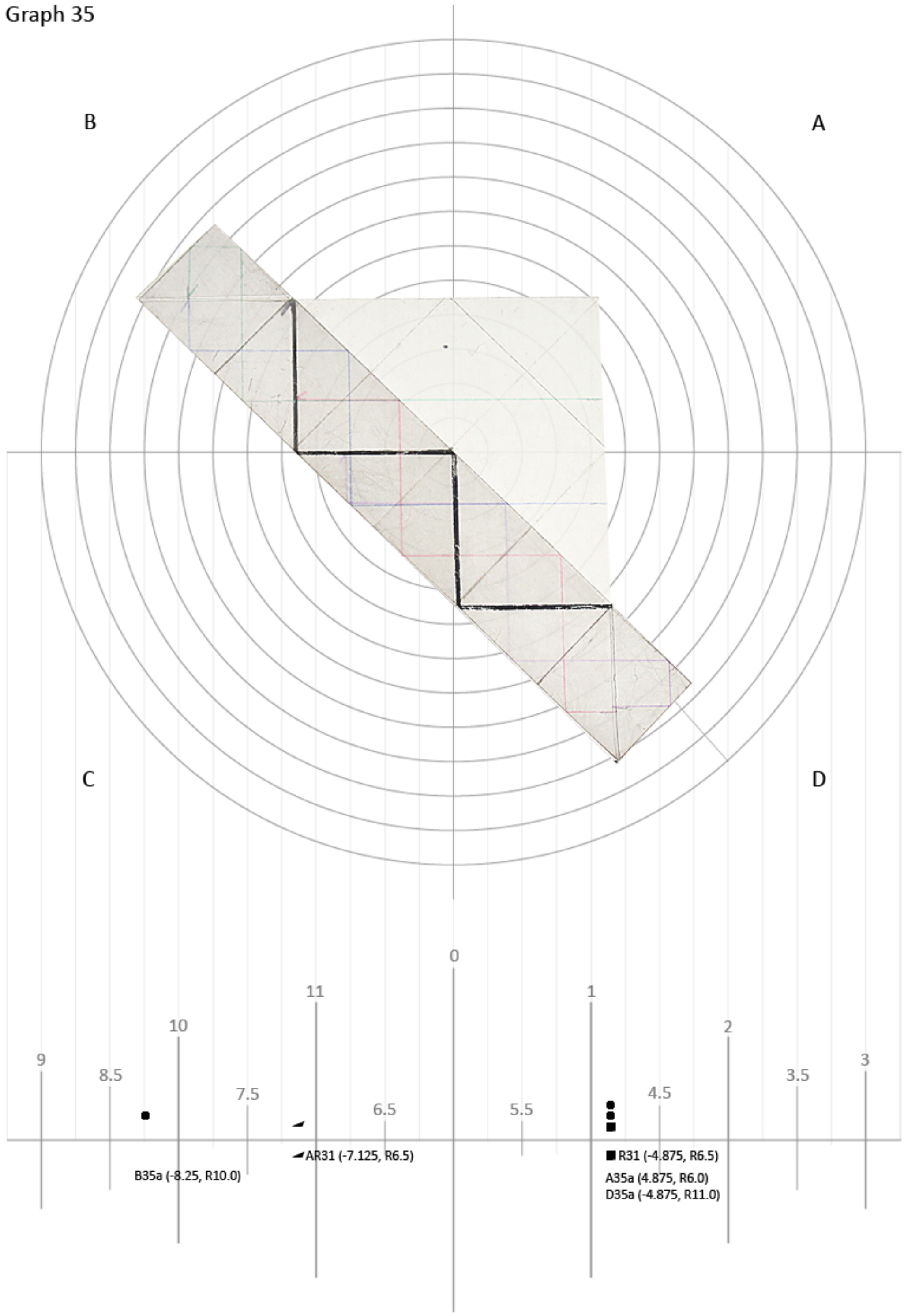
Graph 34





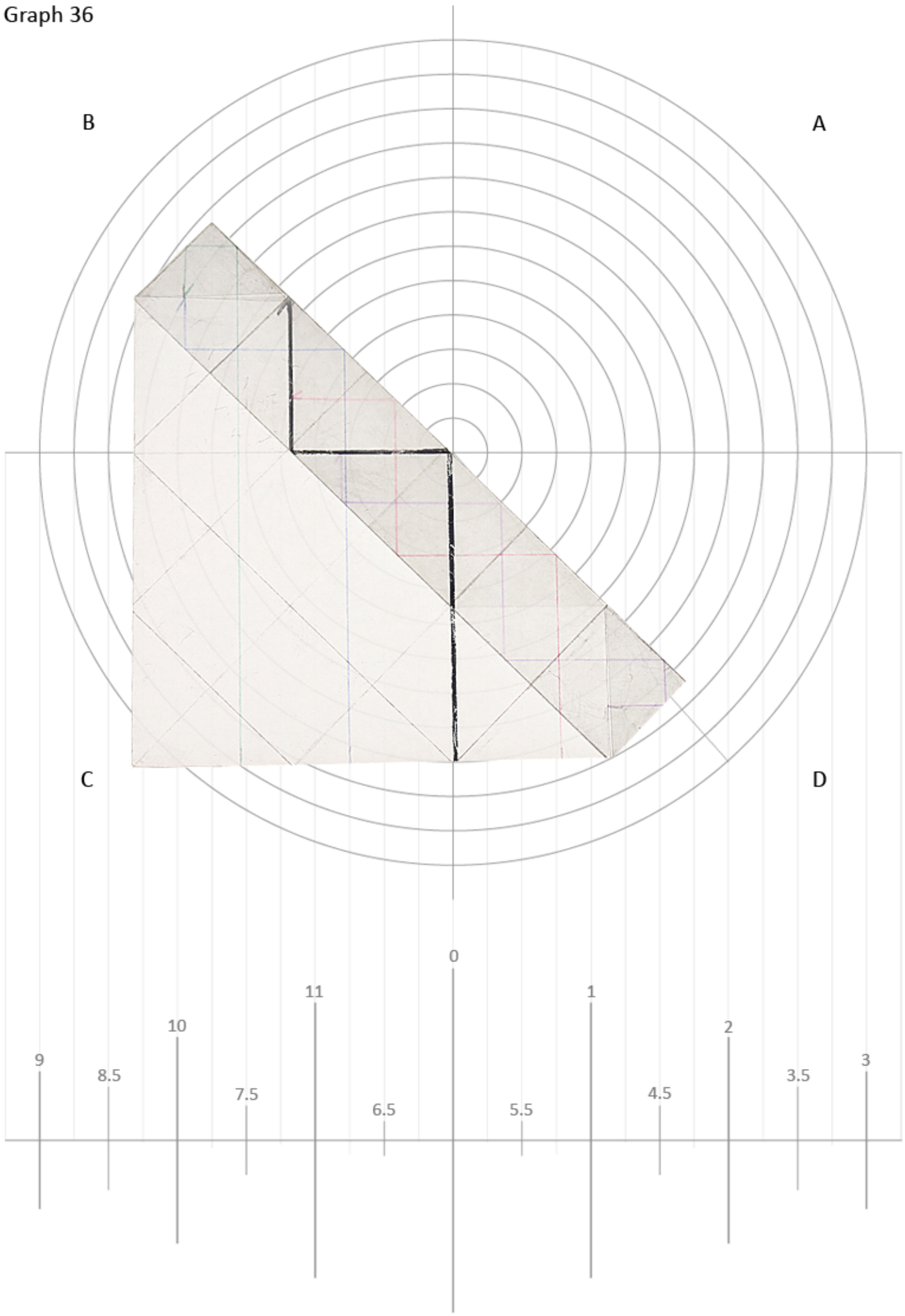
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

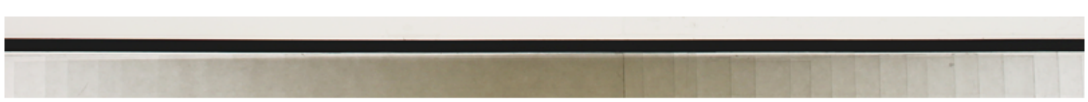
Graph 35



NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

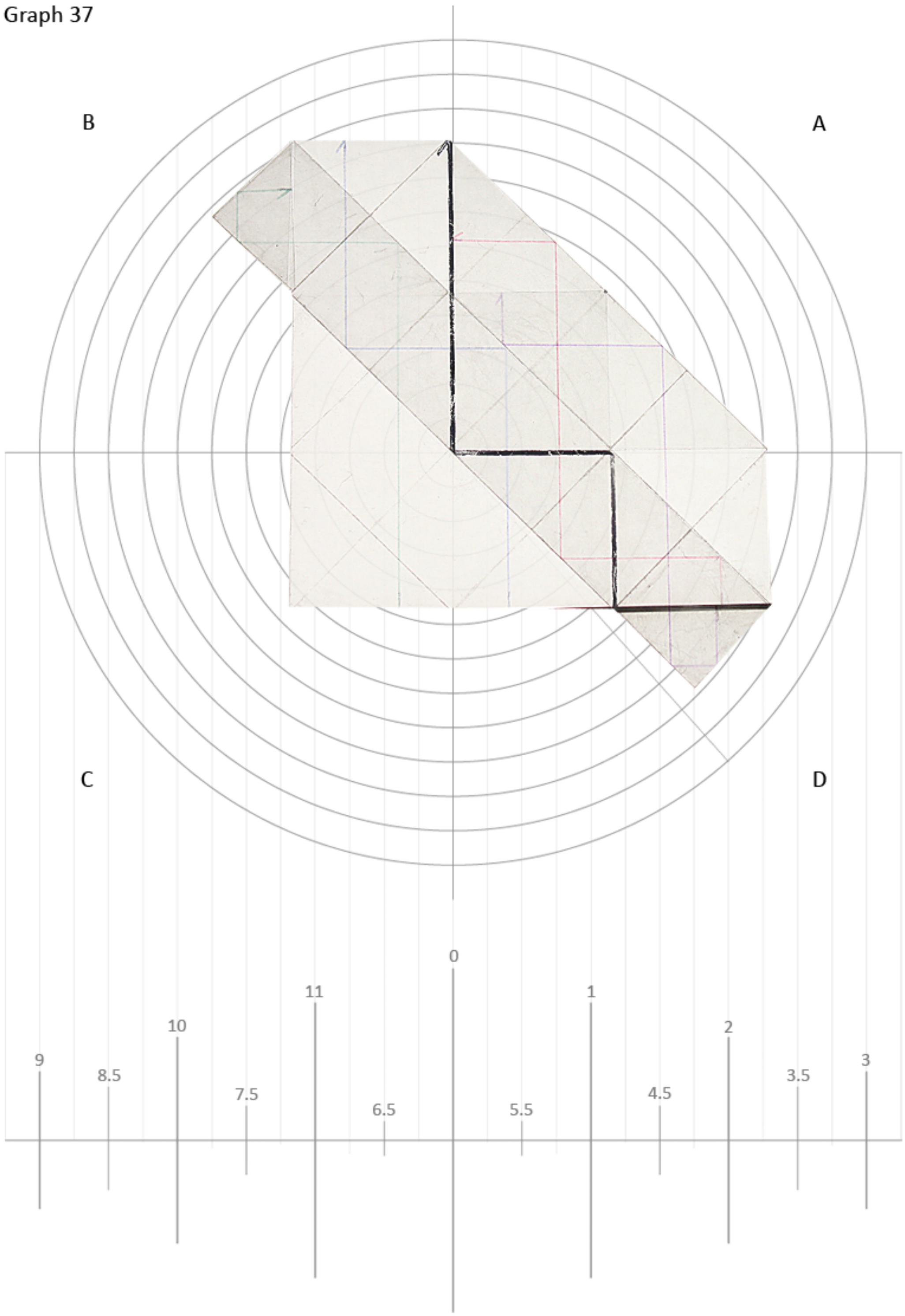
Graph 36

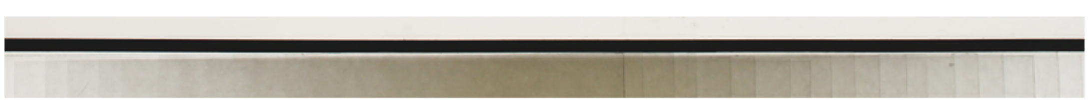




NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

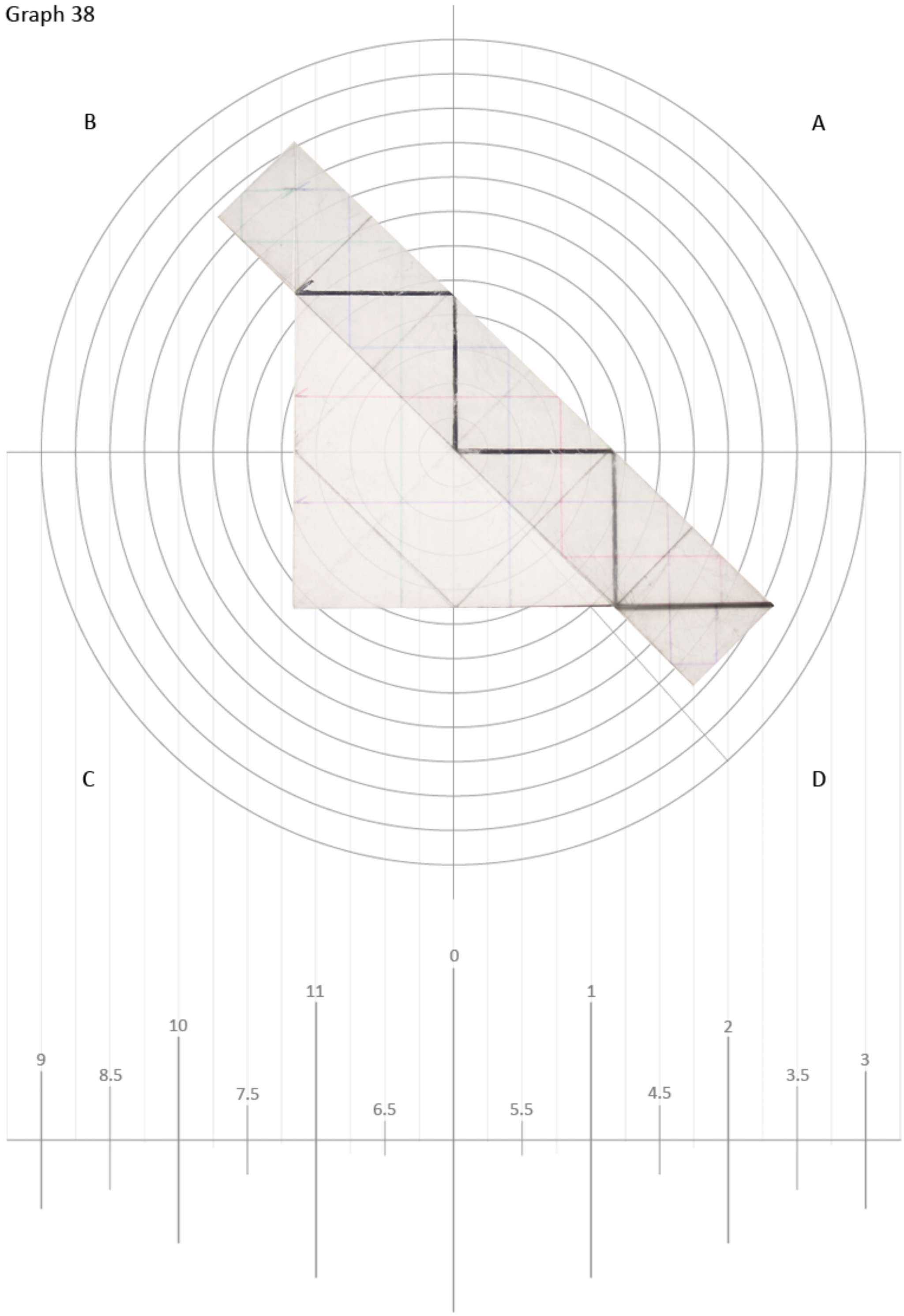
Graph 37





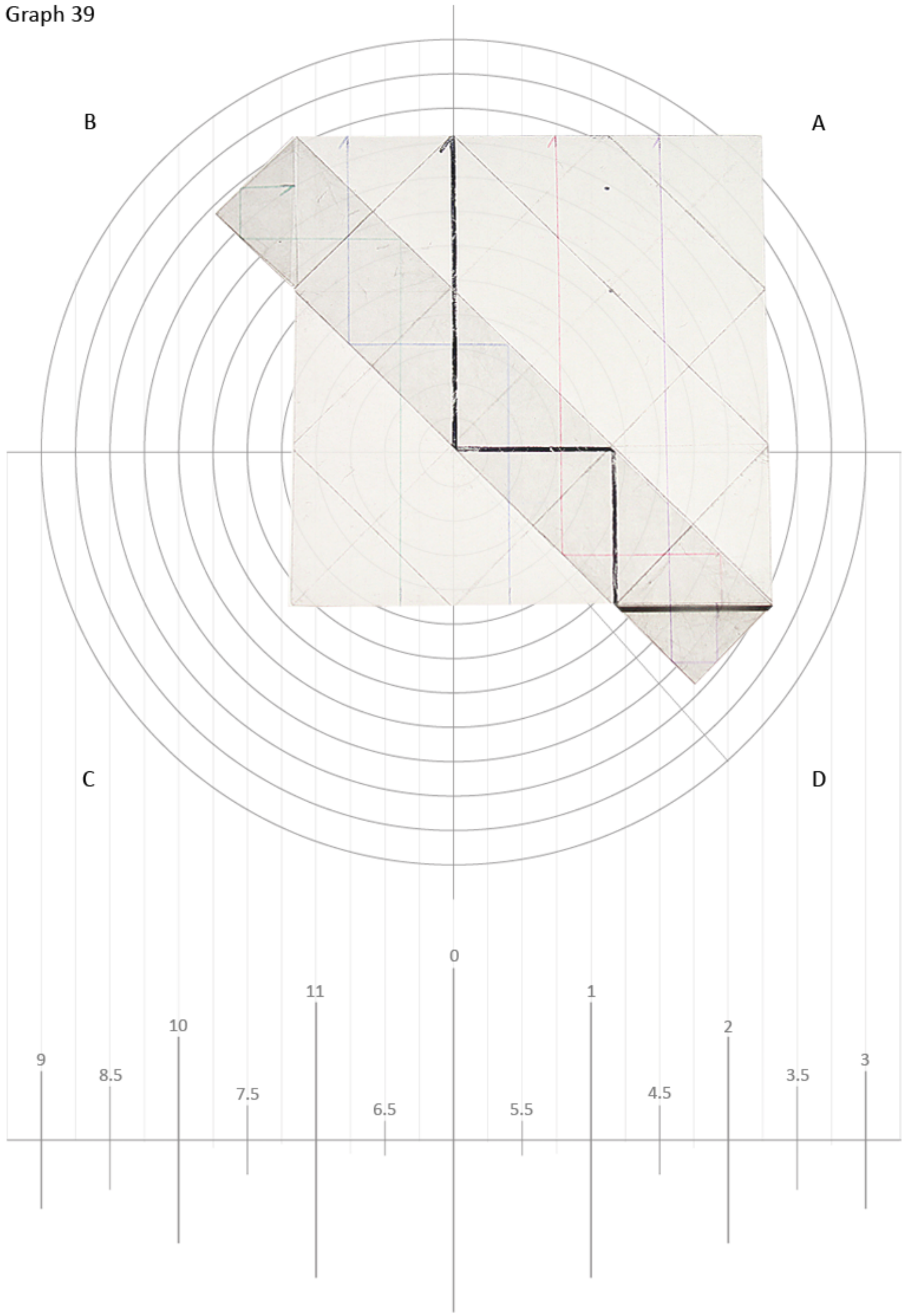
NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

Graph 38



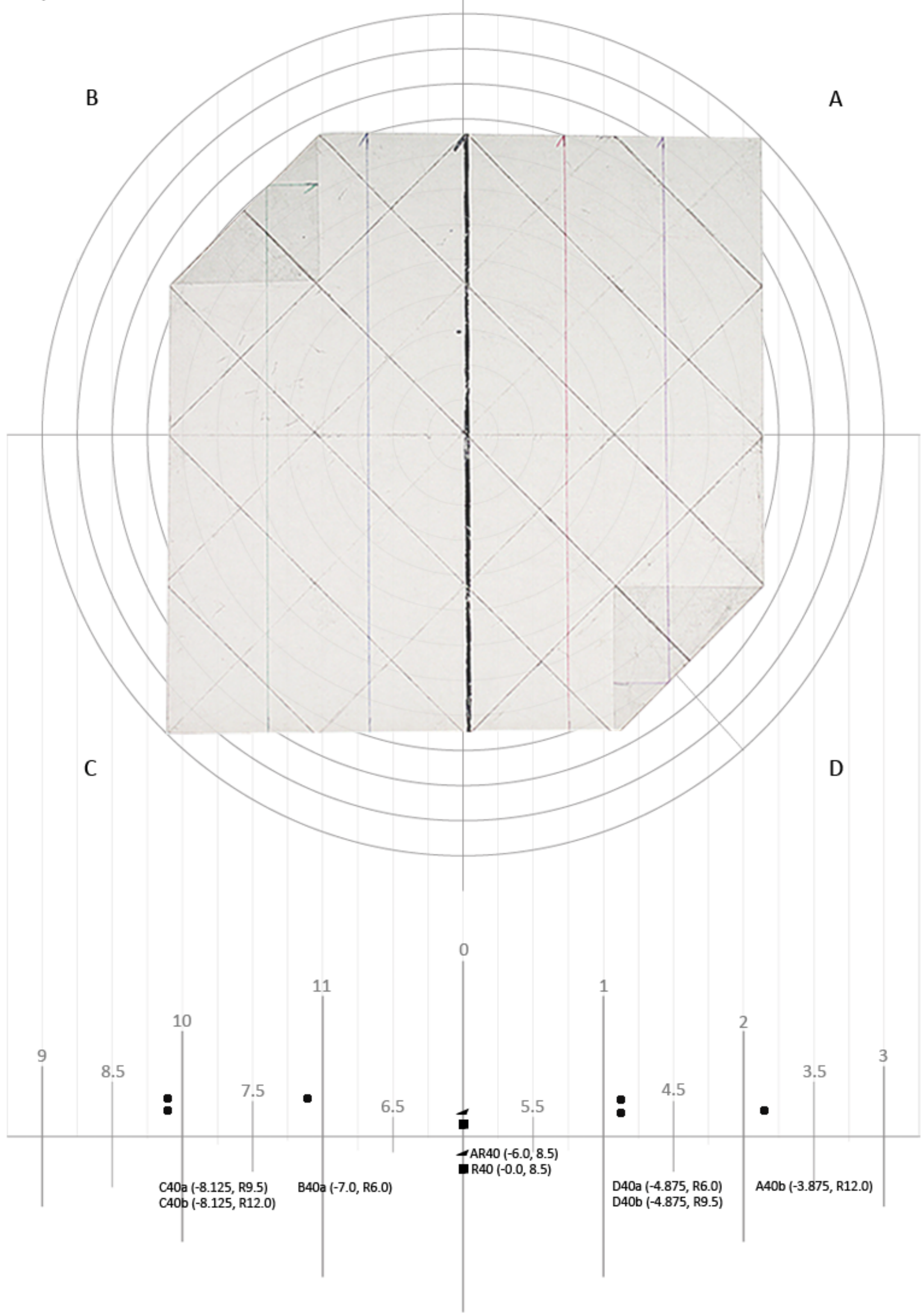
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 39



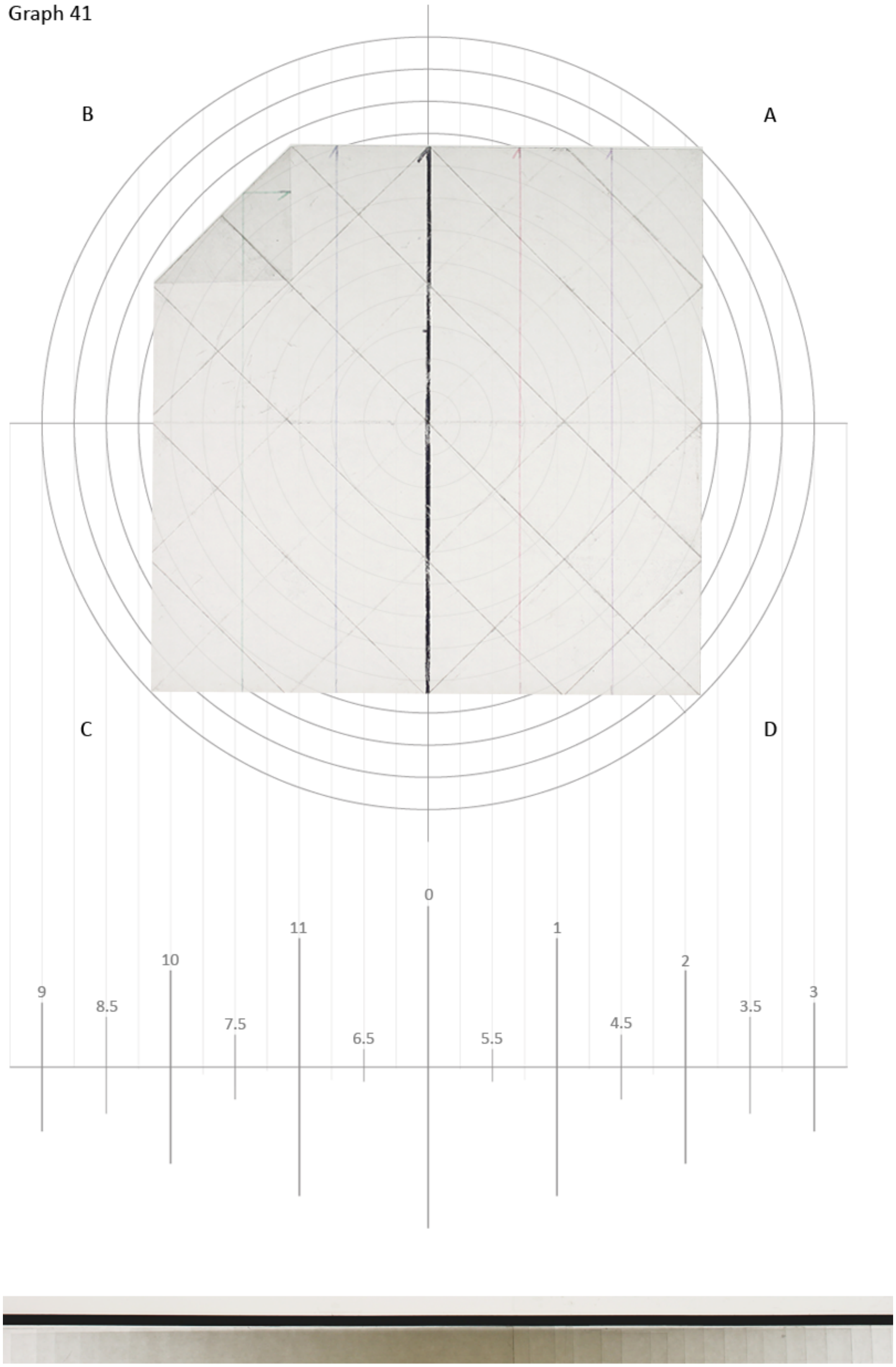
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 40



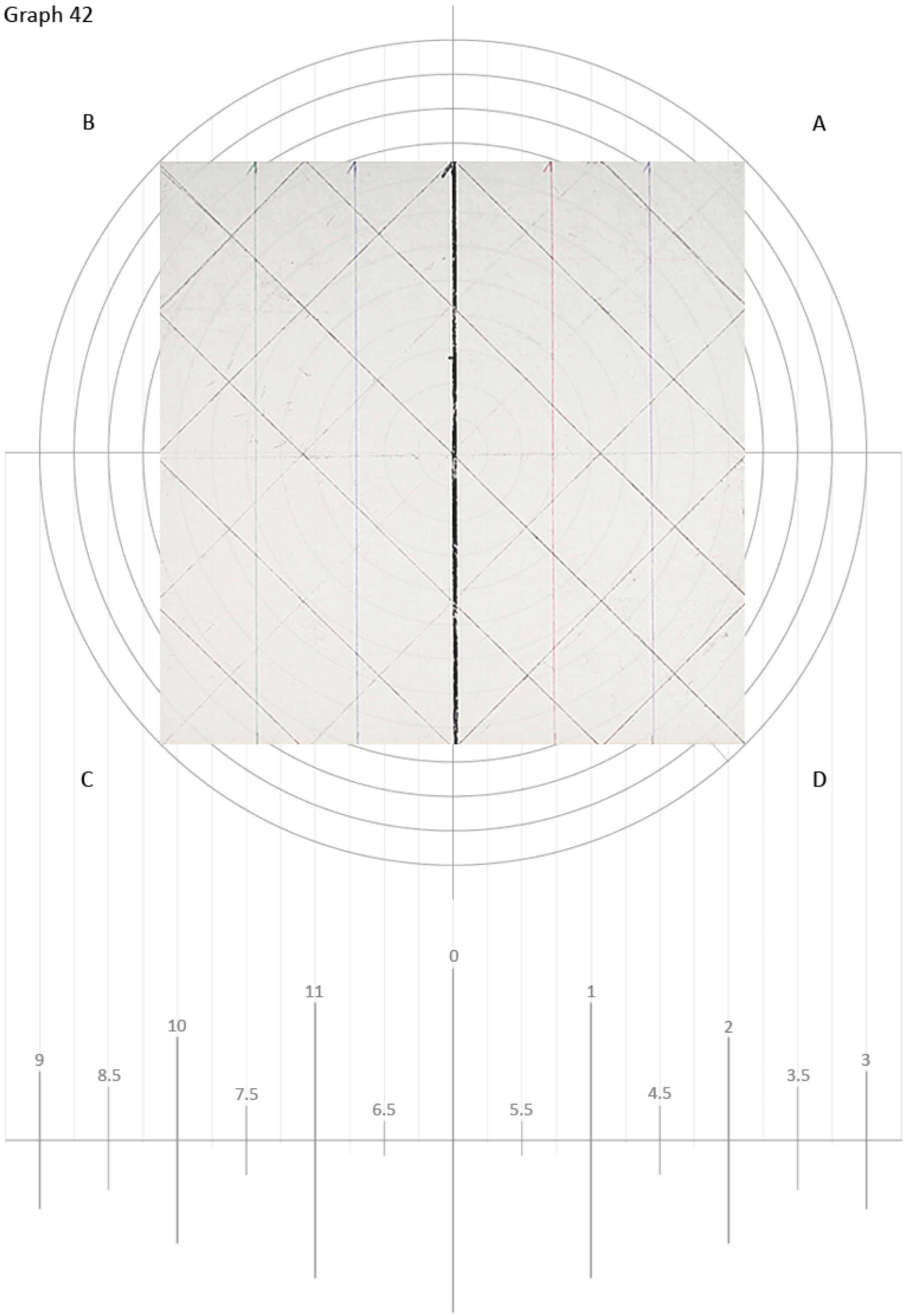
NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

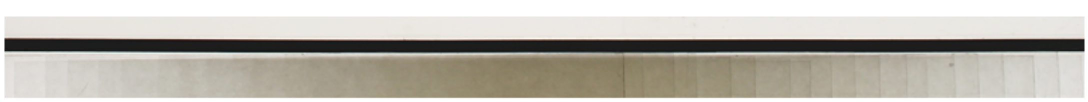
Graph 41



NOTE: ACTUAL SCALE OF PAPER IS 6" X 6"

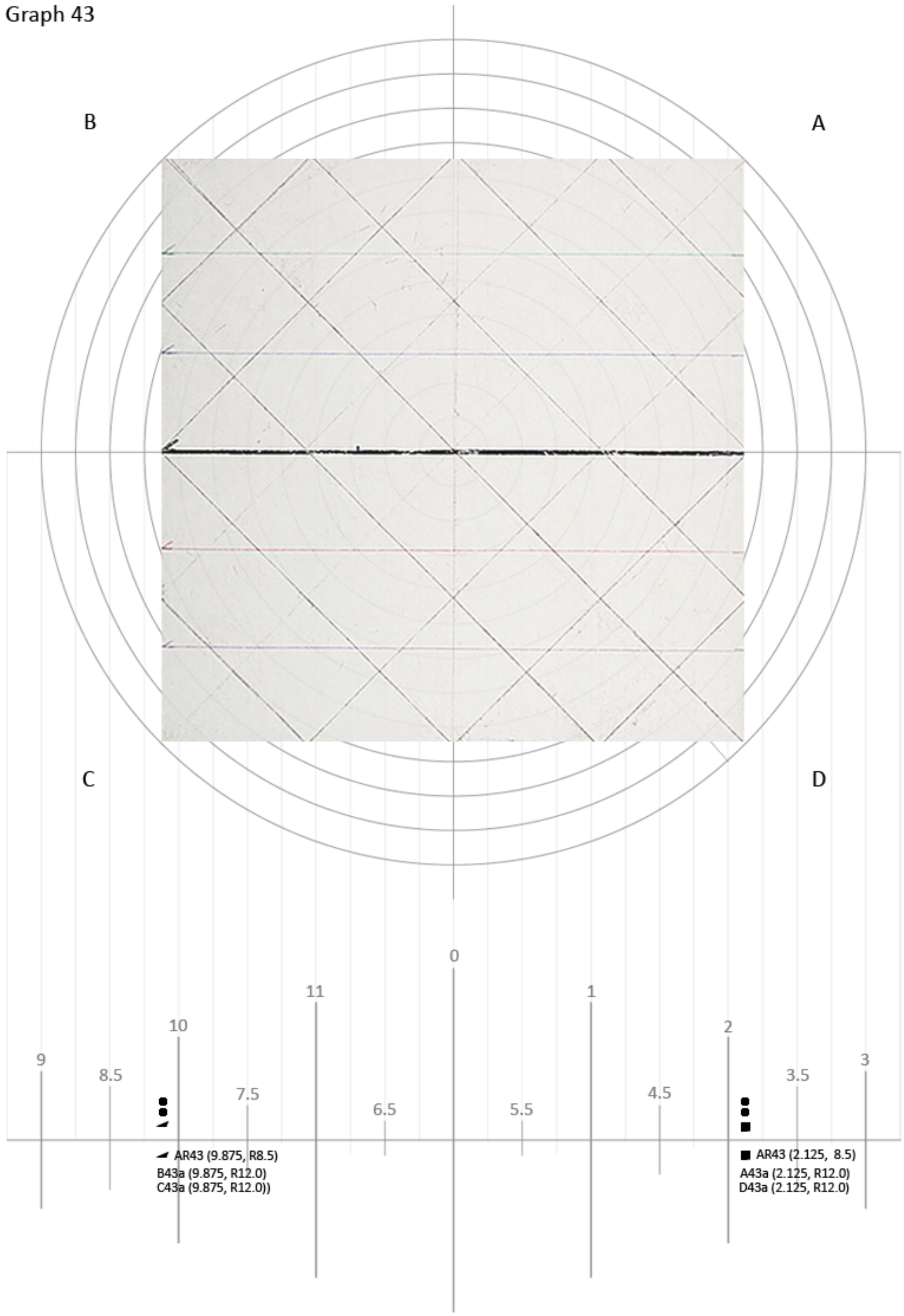
Graph 42





NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

Graph 43



NOTE: ACTUAL SCALE OF PAPER IS 6” X 6”

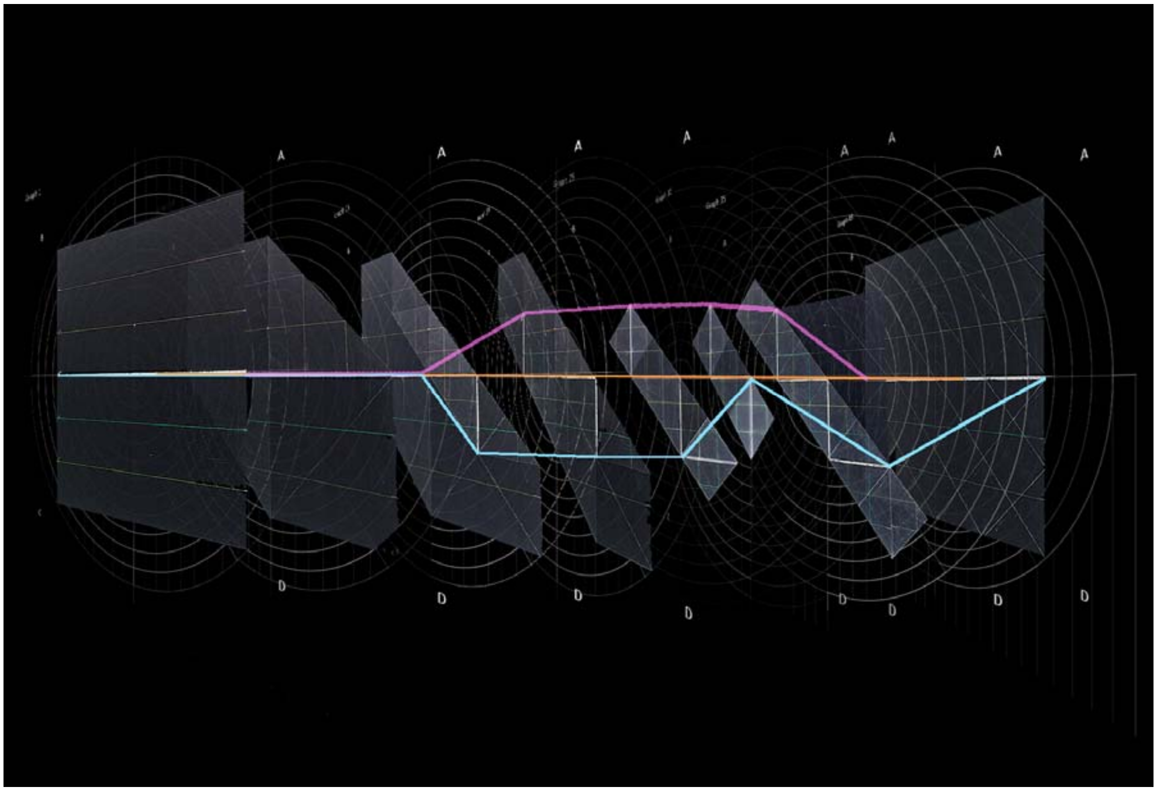


Figure 6.2. Hyperplane Projection: Paper Folding Graph 2 Timeline[96]
The diagram maps out the series of the samples taken from Case Study 2 in 3-Dimensions to study the behavior and structure of space during the folding process. The blue line represents the right end of the primary hyperplane, the pink shows the left end, and the orange line describes the central time axis.

However, in this case it forms a loop of space within the paper folding cycle. Each vertex or point represents a change in information. The unique conclusion from Figures 6.1 and 6.2 occurs when the amount of the negative phase of the paper is in highest proportion to the front. Figure 6.1 generally is an inward folding pattern, whereas Figure 6.2 is based on an accordion type fold. The negative phase folds appear to increase the gap between the endpoints of the hyperplane when it is seen in the timeline or interpolated view as in Figures 6.1 and 6.2. As there are only two case studies presented here it would be recommended that additional ones be studied to examine the phenomenon in terms of its distributive qualities.

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[96] Paper Folding Graph Timeline by author.

SPACE GRAPH

ch7

CONCLUSION

7. CONCLUSION

> I believe we are on the threshold of a new era when the proper understanding of the deep questions of space, as they are embodied in architecture will play a revolutionary role in the way we see the world and will do for the world view of the 21st and 22nd centuries, what physics did for the 19th and 20th.[97]

Architecture is a historically multi-disciplinary practice. Alexander infers that the interrelated issues relating to the spatial and material properties in the study of the built environment have critical components in understanding the nature of space. It is not a discovery solely related to any architect but one which is found in the deeper expressions of its patterns. Alexander suggests that the inanimate environment is related to what is experienced in the progression through space. Space syntax is a specific example of the geometry of spatial arrangement and progression in space with social use. The basic geometric principles used in space syntax are found in examples from the past as well as those proposed for the future. Space is both embodied and embedded in the configurations of the built form through time. The paper folding graphs show spatial transformations through time. It is a way to look deeper into the harmony of the world. In Chapter 4, Lao-Tzu describes how physical objects are dependent on the behavior and operation of space.

This thesis develops a methodology to analyze and identify the operative behavior and structure of space in a 2-Dimensional environment through the paper folding process. Through the research identified in this thesis project it is found that space is not an immaterial entity, but one which is interdependent with the physical (matter) environment. More specifically, it is theorized

[97] Christopher Alexander, *The Nature of Order: The Luminous Ground* (Berkeley: The Center for Environmental Structure, 2004).

that space has structural and operative qualities related to the way it is perceived. The core problem identified by the author is that the information of space is encoded, at least in the paper folding, in relationship to issues of visibility and geometry. In general, space cannot be seen directly but it can possibly be visualized on its effects on the geometry of the folding process. In this thesis, systems theory is used as the basis to understand and compare the relationship between physical form and space through geometry. A comparison is made between systems theory, space syntax, and paper folding to establish a relationship between the physical aspect of the environment and space as an integrated spatial entity.

The experience of the spatial environment and the transformations found in paper folding are time-related, not necessarily dependent, processes. Paper folding is unique because it expresses regular patterns of geometry and spatial information that can be seen on the surface of the paper medium, during the folding process, and interpreted as a matrix of data points. Paper folding is adapted and modified to study any potential behavior using the concept of a hyperplane or mirror-line. This thesis strictly followed several basic premises in paper folding. Namely, that the medium is not cut and cannot penetrate itself. To simplify the issues of complexity of the folding process, only 2-Dimensional folding is considered. The hyperplane functions to cut space without having to physically cut the medium itself. Two case studies are applied to existing methods used in geometry to study transformations in polygons. This led to the development of the concyclic or shell diagrams used in this thesis. The "head-on" view of the graphs where the data points are plotted are a unique interpretation by the author. Using the head-on view the entire plane of the 2-Dimensional folding process is translated into a form where the points are seen as a series of fluctuations. Both the geometry of the paper medium and space itself is also incorporated as data in the graphs. However, due to time requirements and technical complexity, many of the deeper revelations about this method needs to be studied.

The initial goal of this thesis was to identify a more fundamental language or means by which space and the built environment are related. Paper folding is used to focus and simplify ideas of space and physical objects into an interdependent model to study possible behaviors. Following the results from the concyclic graphs, the next immediate step would be to study the data more in-depth as a series of ratios. This is particularly important for spatial analyses such as space syntax because spatial orders are understood also as ratio of the relationships between spaces. The 2-Dimensional format used in this thesis is particularly suited to space syntax because its analyses are also based in 2-D form. However, like the true built environment, which is experienced 3-Dimensionally, paper folding also expresses the complexity of that as well. That would form the long-range goal and application for this topic.

Space should be understood with equal dexterity. Architecture has a historical and experiential library like no other study, where the built form is one of the most persistent qualities of human activity—through space humans and the environment are inextricably linked. Space is as much within the property of buildings as it is with people as a series of integrated relationships. In other words space is its essence. In Chapter 2 Bernard Tschumi described that idea as a fundamental question in architecture. It is suggested from the results that space is somehow linked as a structure with physical form, possibly nested as a specific type of information relating to how dimension is perceived. This doctorate project thesis generalizes the concept of space within the context of paper folding as a spatial organizing tool. Paper folding exhibits the ability to measure and create spatial patterns, or forms, and can also be simultaneously manipulated much as a conceptual architectural spatial model. When paper folding is used in a more defined manner as in this thesis its range of its application appears much broader. It appears that the paper seems not to be folding in space, but folding *with* it. Space is more than a collection of objects and distributions between them, but also one of pure event.

While the paper folding graphs each have their own unique pattern or fingerprint depending on what type of folding is used, they generally are all created from a single folding principle. The single fold initiates the change in the state of the paper from an undifferentiated form to a set of sequential layers and interrelated divisions as explained in Chapter 3. However the full nature of the folding process in paper folding space is not necessarily compressed, but takes on a more subtle, precise, and efficient form. Space does not have to be a substance as the material or physical environment is known to be. In fact, a general review of the literature does not make a suggestion that it is a substance of any sort. So, if space is not a substance, it could be a projection of some sort. If the hyperplane or mirror-line is seen as the worm in the quote by Emerson at the beginning of the thesis, the reader can imagine that the paper folding graphs follow a similar journey--the mirror-line being the worm reflecting the image of what a person perceives as spatial form as one moves through the spatial environment. The worm can be followed like a string through a maze. Space embodies what is known...and what has yet to be realized.

8. FIGURES

[98] Graph by author.
[99] Fold and photograph by author, 2012.
[100] Kunihiko Kasahara, *Origami Omnibu*s*, (Tokyo:* Japan Publications, 2000).
[101] Robert Lang, “Treemaker Software Crease Patterns,” last modified December 2011, http://www.langorigami.com/science/computational/treemaker/treemaker.php.
[102] Art and photo by author.
[103] Art and photo by author.
[104] Cornelius Van de Ven, *Space in Architectur, (Assen:* Van Gorcum, 1977).
[105] Folds and photo by author.
[106] Diagram by author.
[107] Graph redrawn by author. Original source: Bill Hillier, *Space Syntax* (London: University College, 1976.
[108] Diagram courtesy of author.
[109] Diagram courtesy of author.
[110] Diagram courtesy of author.
[111] Diagram courtesy of author.

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[112] Diagram courtesy of author.
[113] Bill Hillier, *Space is the Machine* (London: University College London, 2007).
[114] Fold and diagram courtesy of author. Underlying image: Robert J. Lang, *Origami Design Secrets* (Massachusetts: A.K. Peters, 2003), 375-408.
[115] Robert J. Lang, *Origami Design Secrets* (Massachusetts: A.K. Peters, 2003), 375-408.
[116] Robert J. Lang, *Origami Design Secrets* (Massachusetts: A.K. Peters, 2003), 375-408.
[117] Alasdair Turner, “UCL Depthmap 7: Axial Line Analysis: *Version 7.12.00c”* (London: University College London, 2012).
[118] Ben Doherty. 2011.
[119] Folds and photo by author.
[120] Folds and photo by author.
[121] Folds and photo by author.
[122] Folds and photo by author.
[123] Folds and photo by author.
[124] Folds and photo by author.
[125] Folds and photo by author.
[126] Folds and photo by author.
[127] Folds and photo by author.

[128] Robert J. Lang, "Huzita-Justin Axioms," Last modified November 11, 2009, http://www.langorigami.com/science/math/hja/hja.php.
[129] Graphs by author.
[130] Graph by author.
[131] Paper Folding Graph Timeline by author.
[132] Paper Folding Graph Timeline by author.

9. GLOSSARY

Abstract Paper Space: The environment where the folding properties of paper are described through geometry, graphs, and other abstract diagrammatic methods of analysis. Paper space is contrasted with *physical paper space*, which refers to the physical environment where paper is folded by the user.

Accuracy: A characteristic of a measurement having a low systemic error--that is, not consistently over- or underestimating a value.

Axial Line Analysis: A method used in space syntax to analyze the linear relationships between Spatial points in the built environment.

Built Environment: The condition whereby the physical environment is manipulated by human intervention in response to biological, social, and economic impulse at various levels of scale and complexity. Architecture is a microcosm of the built environment where its activities are a specific study and practice of built forms.

Circle-River Method (CRM): A folding technique based on crease and area fold patterns developed by Robert Lang.

Concyclic Diagram: A method used in geometry to plot polygons within a circumscribed circle.

Cyclic Quadrilateral: A cyclic quadrilateral is any general polygon used in the concyclic diagram.

Depthmap: Depthmap is a specific technique used in space syntax analysis to map out the layout of a series of building spaces in the form of a tree or justified graph and is based on Graph Theory.

Expectancy Bias: (An observation bias) Seeing what an observer wants to see.

Graph Theory: A method of geometry to study the collection of a range of objects.

Hyperplane: A method of geometry used to divide any dimensional surface abstractly.

Index Fold: The initial fold in the folding process.

Isovist: An isovist is the viewable area within a space such as in a building.

Mirror-Line: A mirror-line is analogous to a hyperplane.

Paper Folding: A synonymous term for 'origami'. It is the act of making successive layers. Paper folding is the systematic grouping of one or more layers of paper over another to form, or represent an arbitrary object either in two or three-dimensions. Paper folding is inherently organized through geometric, mathematical, and spatial principles.

Observer Bias: The act of observation may cause the subject of observation to change behavior, affecting results. The simplest solution is to keep the observations secret.

Physical Paper Space: Refers to the physical environment where paper is folded by the user.

Precision: A characteristic of a measurement having a low random error; highly consistent results even if they are far from the true value.

Profile Graphs: These represent the 'side' view of the shell graphs or diagrams.

Random Error: An error that is not predictable for individual observations; not consistent or dependent on known variables (although such error follow the rules of probability in large groups).

Selection Bias: (An observation bias) Even with attempted randomness, inadvertent non-randomness may occur.

Shells: Technically referred to as concyclic diagrams or cyclic quadrilaterals in geometry. The information created by the folding process as it is diagrammatically graphed along a circle. These graphs represent the 'overhead' view of the folds.

Space Syntax: A systematic method of spatial analysis used in architecture to study the behavior of the built environment and social behavior.

Systemic Error/Bias: An inherent tendency of a measurement process to favor a particular outcome; a consistent bias.

TRIZ: *Teoriya **R**esheniya **I**zobretatelskikh **Z**adatch)* or the Theory of Inventive Problem Solving which is a type of applied methodology used for technological development.

Undifferentiated Folded Paper Space (or Undifferentiated Paper Space)**:** The condition of a sheet of paper before any physical folding has occurred, or given state, such as those found cases of folding with existing mappings of puzzle grids, roads ,and space syntax analyses.

Note: Terminology and definitions of observation bias and types of error are referenced from *How to Measure Anything*.[133] The terminology in italicized form is those of the author and specifically created for this thesis.

[133] Douglas W. Hubbard, *How to Measure Anything: Finding the Value of Intangibles in Busines*.(New Jersey: John Wiley & Sons, 2010).

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